3

Vectors

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q3.1 No. The sum of two vectors can only be zero if they are in opposite directions and have the same magnitude. If you walk 10 meters north and then 6 meters south, you won't end up where you started.

Q3.2 No, the magnitude of the displacement is always less than or equal to the distance traveled. If two displacements in the same direction are added, then the magnitude of their sum will be equal to the distance traveled. Two vectors in any other orientation will give a displacement less than the distance traveled. If you first walk 3 meters east, and then 4 meters south, you will have walked a total distance of 7 meters, but you will only be 5 meters from your starting point.

Q3.3 The largest possible magnitude of $\mathbf{R} = \mathbf{A} + \mathbf{B}$ is 7 units, found when **A** and **B** point in the same direction. The smallest magnitude of $\mathbf{R} = \mathbf{A} + \mathbf{B}$ is 3 units, found when **A** and **B** have opposite directions.

Q3.4 Only force and velocity are vectors. None of the other quantities requires a direction to be described.

Q3.5 If the direction-angle of **A** is between 180 degrees and 270 degrees, its components are both negative. If a vector is in the second quadrant or the fourth quadrant, its components have opposite signs.

Q3.6 The book's displacement is zero, as it ends up at the point from which it started. The distance traveled is 6.0 meters.

Q3.7 85 miles. The magnitude of the displacement is the distance from the starting point, the 260-mile mark, to the ending point, the 175-mile mark.

Q3.8 Vectors **A** and **B** are perpendicular to each other.

Q3.9 No, the magnitude of a vector is always positive. A minus sign in a vector only indicates direction, not magnitude.

Q3.10 Any vector that points along a line at 45° to the x and y axes has components equal in magnitude.

Q3.11 $A_x = B_x$ and $A_y = B_y$.

Q3.12 Addition of a vector to a scalar is not defined. Think of apples and oranges.

Q3.13 One difficulty arises in determining the individual components. The relationships between a vector and its components such as $A_x = A\cos\theta$, are based on right-triangle trigonometry. Another problem would be in determining the magnitude or the direction of a vector from its components. Again, $A = \sqrt{A_x^2 + A_y^2}$ only holds true if the two component vectors, $\mathbf{A}_x$ and $\mathbf{A}_y$, are perpendicular.

Q3.14 If the direction of a vector is specified by giving the angle of the vector measured clockwise from the positive y-axis, then the x-component of the vector is equal to the sine of the angle multiplied by the magnitude of the vector.

SOLUTIONS TO PROBLEMS

Section 3.1 **Coordinate Systems**

P3.1 $x = r\cos\theta = (5.50\text{ m})\cos 240° = (5.50\text{ m})(-0.5) = \boxed{-2.75\text{ m}}$

$y = r\sin\theta = (5.50\text{ m})\sin 240° = (5.50\text{ m})(-0.866) = \boxed{-4.76\text{ m}}$

P3.2 (a) $x = r\cos\theta$ and $y = r\sin\theta$, therefore

$x_1 = (2.50\text{ m})\cos 30.0°$, $y_1 = (2.50\text{ m})\sin 30.0°$, and

$(x_1, y_1) = \boxed{(2.17,\ 1.25)\text{ m}}$

$x_2 = (3.80\text{ m})\cos 120°$, $y_2 = (3.80\text{ m})\sin 120°$, and

$(x_2, y_2) = \boxed{(-1.90,\ 3.29)\text{ m}}$.

(b) $d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{16.6 + 4.16} = \boxed{4.55\text{ m}}$

P3.3 The x distance out to the fly is 2.00 m and the y distance up to the fly is 1.00 m.

(a) We can use the Pythagorean theorem to find the distance from the origin to the fly.

$$\text{distance} = \sqrt{x^2 + y^2} = \sqrt{(2.00\text{ m})^2 + (1.00\text{ m})^2} = \sqrt{5.00\text{ m}^2} = \boxed{2.24\text{ m}}$$

(b) $\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.6°$; $\mathbf{r} = \boxed{2.24\text{ m},\ 26.6°}$

P3.4 (a) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2.00 - [-3.00])^2 + (-4.00 - 3.00)^2}$

$d = \sqrt{25.0 + 49.0} = \boxed{8.60 \text{ m}}$

(b) $r_1 = \sqrt{(2.00)^2 + (-4.00)^2} = \sqrt{20.0} = \boxed{4.47 \text{ m}}$

$\theta_1 = \tan^{-1}\left(-\frac{4.00}{2.00}\right) = \boxed{-63.4°}$

$r_2 = \sqrt{(-3.00)^2 + (3.00)^2} = \sqrt{18.0} = \boxed{4.24 \text{ m}}$

$\theta_2 = \boxed{135°}$ measured from the $+x$ axis.

P3.5 We have $2.00 = r\cos 30.0°$

$$r = \frac{2.00}{\cos 30.0°} = \boxed{2.31}$$

and $y = r\sin 30.0° = 2.31\sin 30.0° = \boxed{1.15}$.

P3.6 We have $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$.

(a) The radius for this new point is

$$\sqrt{(-x)^2 + y^2} = \sqrt{x^2 + y^2} = \boxed{r}$$

and its angle is

$$\tan^{-1}\left(\frac{y}{-x}\right) = \boxed{180° - \theta}.$$

(b) $\sqrt{(-2x)^2 + (-2y)^2} = \boxed{2r}$. This point is in the third quadrant if (x, y) is in the first quadrant or in the fourth quadrant if (x, y) is in the second quadrant. It is at an angle of $\boxed{180° + \theta}$.

(c) $\sqrt{(3x)^2 + (-3y)^2} = \boxed{3r}$. This point is in the fourth quadrant if (x, y) is in the first quadrant or in the third quadrant if (x, y) is in the second quadrant. It is at an angle of $\boxed{-\theta}$.

Section 3.2 **Vector and Scalar Quantities**

Section 3.3 **Some Properties of Vectors**

P3.7 $\tan 35.0° = \dfrac{x}{100 \text{ m}}$

$$x = (100 \text{ m})\tan 35.0° = \boxed{70.0 \text{ m}}$$

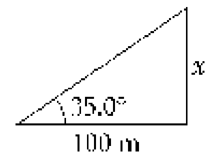


FIG. P3.7

P3.8 $R = \boxed{14 \text{ km}}$

$\theta = \boxed{65° \text{ N of E}}$

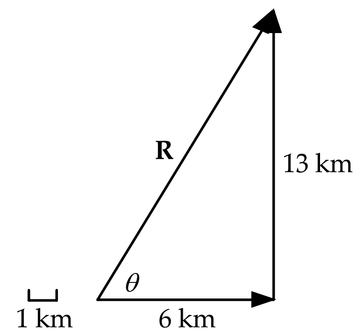


FIG. P3.8

P3.9 $-\mathbf{R} = \boxed{310 \text{ km at } 57° \text{ S of W}}$

(Scale: 1 unit = 20 km)

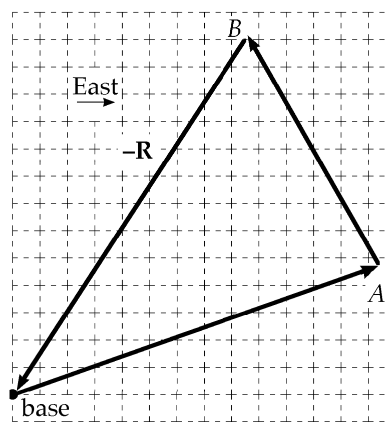


FIG. P3.9

P3.10 (a) Using graphical methods, place the tail of vector **B** at the head of vector **A**. The new vector $\mathbf{A}+\mathbf{B}$ has a magnitude of $\boxed{6.1 \text{ at } 112°}$ from the x-axis.

(b) The vector difference $\mathbf{A}-\mathbf{B}$ is found by placing the negative of vector **B** at the head of vector **A**. The resultant vector $\mathbf{A}-\mathbf{B}$ has magnitude $\boxed{14.8}$ units at an $\boxed{\text{angle of } 22°}$ from the $+\ x$-axis.

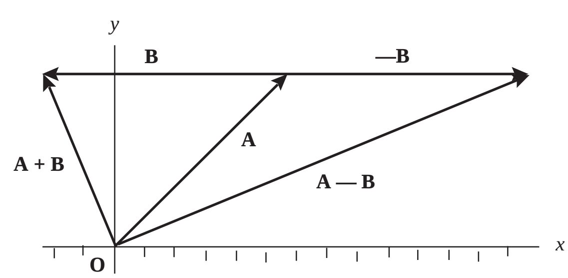


FIG. P3.10

P3.11 (a) $|\mathbf{d}| = |-10.0\hat{\mathbf{i}}| = \boxed{10.0\text{ m}}$ since the displacement is in a straight line from point A to point B.

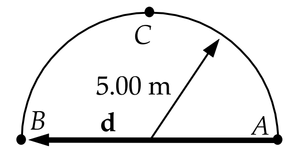


FIG. P3.11

(b) The actual distance skated is not equal to the straight-line displacement. The distance follows the curved path of the semi-circle (*ACB*).

$$s = \frac{1}{2}(2\pi r) = 5\pi = \boxed{15.7\text{ m}}$$

(c) If the circle is complete, **d** begins and ends at point **A**. Hence, $|\mathbf{d}| = \boxed{0}$.

P3.12 Find the resultant $\mathbf{F}_1 + \mathbf{F}_2$ graphically by placing the tail of $\mathbf{F}_2$ at the head of $\mathbf{F}_1$. The resultant force vector $\mathbf{F}_1 + \mathbf{F}_2$ is of magnitude $\boxed{9.5\text{ N}}$ and at an angle of $\boxed{57° \text{ above the } x\text{-axis}}$.

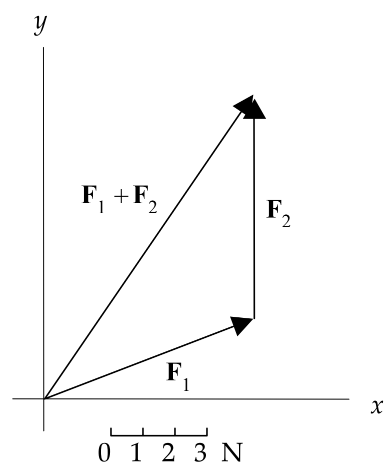


FIG. P3.12

P3.13 (a) The large majority of people are standing or sitting at this hour. Their instantaneous foot-to-head vectors have upward vertical components on the order of 1 m and randomly oriented horizontal components. The citywide sum will be $\boxed{\sim 10^5 \text{ m upward}}$.

(b) Most people are lying in bed early Saturday morning. We suppose their beds are oriented north, south, east, west quite at random. Then the horizontal component of their total vector height is very nearly zero. If their compressed pillows give their height vectors vertical components averaging 3 cm, and if one-tenth of one percent of the population are on-duty nurses or police officers, we estimate the total vector height as $\sim 10^5(0.03\text{ m}) + 10^2(1\text{ m})$ $\boxed{\sim 10^3 \text{ m upward}}$.

P3.14 Your sketch should be drawn to scale, and should look somewhat like that pictured to the right. The angle from the westward direction, θ, can be measured to be $\boxed{4° \text{ N of W}}$, and the distance R from the sketch can be converted according to the scale to be $\boxed{7.9 \text{ m}}$.

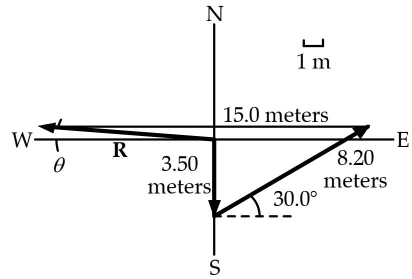


FIG. P3.14

P3.15 To find these vector expressions graphically, we draw each set of vectors. Measurements of the results are taken using a ruler and protractor. (Scale: 1 unit = 0.5 m)

(a) $\mathbf{A} + \mathbf{B}$ = 5.2 m at 60°

(b) $\mathbf{A} - \mathbf{B}$ = 3.0 m at 330°

(c) $\mathbf{B} - \mathbf{A}$ = 3.0 m at 150°

(d) $\mathbf{A} - 2\mathbf{B}$ = 5.2 m at 300°.

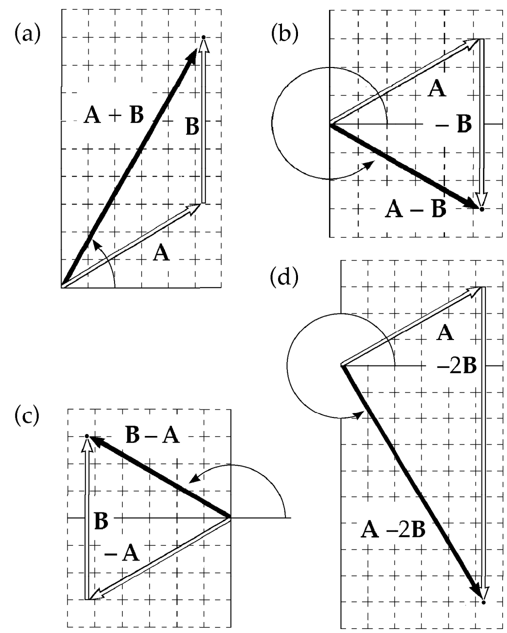


FIG. P3.15

***P3.16** The three diagrams shown below represent the graphical solutions for the three vector sums: $\mathbf{R}_1 = \mathbf{A} + \mathbf{B} + \mathbf{C}$, $\mathbf{R}_2 = \mathbf{B} + \mathbf{C} + \mathbf{A}$, and $\mathbf{R}_3 = \mathbf{C} + \mathbf{B} + \mathbf{A}$. You should observe that $\mathbf{R}_1 = \mathbf{R}_2 = \mathbf{R}_3$, illustrating that the sum of a set of vectors is not affected by the order in which the vectors are added.

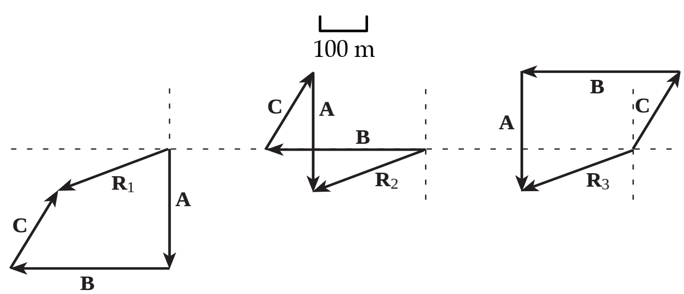
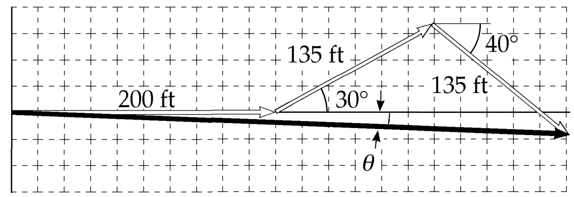


FIG. P3.16

P3.17 The scale drawing for the graphical solution should be similar to the figure to the right. The magnitude and direction of the final displacement from the starting point are obtained by measuring d and θ on the drawing and applying the scale factor used in making the drawing. The results should be

$$\boxed{d = 420 \text{ ft and } \theta = -3°}$$



(Scale: 1 unit = 20 ft)

FIG. P3.17

Section 3.4 **Components of a Vector and Unit Vectors**

P3.18 Coordinates of the super-hero are:

$$x = (100 \text{ m})\cos(-30.0°) = \boxed{86.6 \text{ m}}$$

$$y = (100 \text{ m})\sin(-30.0°) = \boxed{-50.0 \text{ m}}$$

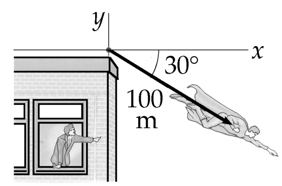


FIG. P3.18

P3.19 $A_x = -25.0$

$A_y = 40.0$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25.0)^2 + (40.0)^2} = \boxed{47.2 \text{ units}}$$

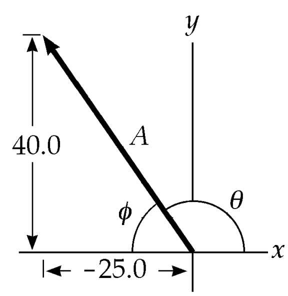


FIG. P3.19

We observe that

$$\tan\phi = \frac{|A_y|}{|A_x|}.$$

So

$$\phi = \tan^{-1}\left(\frac{A_y}{|A_x|}\right) = \tan\frac{40.0}{25.0} = \tan^{-1}(1.60) = 58.0°.$$

The diagram shows that the angle from the $+x$ axis can be found by subtracting from 180°:

$$\theta = 180° - 58° = \boxed{122°}.$$

P3.20 The person would have to walk $3.10\sin(25.0°) = \boxed{1.31 \text{ km north}}$, and

$$3.10\cos(25.0°) = \boxed{2.81 \text{ km east}}.$$

P3.21 $x = r\cos\theta$ and $y = r\sin\theta$, therefore:

(a) $x = 12.8\cos 150°$, $y = 12.8\sin 150°$, and $(x,\ y) = \left(-11.1\hat{\mathbf{i}} + 6.40\hat{\mathbf{j}}\right)$ m

(b) $x = 3.30\cos 60.0°$, $y = 3.30\sin 60.0°$, and $(x,\ y) = \left(1.65\hat{\mathbf{i}} + 2.86\hat{\mathbf{j}}\right)$ cm

(c) $x = 22.0\cos 215°$, $y = 22.0\sin 215°$, and $(x,\ y) = \left(-18.0\hat{\mathbf{i}} - 12.6\hat{\mathbf{j}}\right)$ in

P3.22 $x = d\cos\theta = (50.0\text{ m})\cos(120) = -25.0\text{ m}$
$y = d\sin\theta = (50.0\text{ m})\sin(120) = 43.3\text{ m}$
$\mathbf{d} = \boxed{(-25.0\text{ m})\hat{\mathbf{i}} + (43.3\text{ m})\hat{\mathbf{j}}}$

***P3.23** (a) Her net x (east-west) displacement is $-3.00 + 0 + 6.00 = +3.00$ blocks, while her net y (north-south) displacement is $0 + 4.00 + 0 = +4.00$ blocks. The magnitude of the resultant displacement is

$$R = \sqrt{(x_{\text{net}})^2 + (y_{\text{net}})^2} = \sqrt{(3.00)^2 + (4.00)^2} = 5.00\text{ blocks}$$

and the angle the resultant makes with the x-axis (eastward direction) is

$$\theta = \tan^{-1}\left(\frac{4.00}{3.00}\right) = \tan^{-1}(1.33) = 53.1°.$$

The resultant displacement is then $\boxed{5.00\text{ blocks at }53.1°\text{ N of E}}$.

(b) The total distance traveled is $3.00 + 4.00 + 6.00 = \boxed{13.0\text{ blocks}}$.

***P3.24** Let $\hat{\mathbf{i}} = \text{east}$ and $\hat{\mathbf{j}} = \text{north}$. The unicyclist's displacement is, in meters

$$280\hat{\mathbf{j}} + 220\hat{\mathbf{i}} + 360\hat{\mathbf{j}} - 300\hat{\mathbf{i}} - 120\hat{\mathbf{j}} + 60\hat{\mathbf{i}} - 40\hat{\mathbf{j}} - 90\hat{\mathbf{i}} + 70\hat{\mathbf{j}}.$$

$$\mathbf{R} = -110\hat{\mathbf{i}} + 550\hat{\mathbf{j}}$$
$$= \sqrt{(110\text{ m})^2 + (550\text{ m})^2}\ \text{at}\ \tan^{-1}\frac{110\text{ m}}{550\text{ m}}\ \text{west of north}$$
$$= 561\text{ m at }11.3°\text{ west of north}.$$

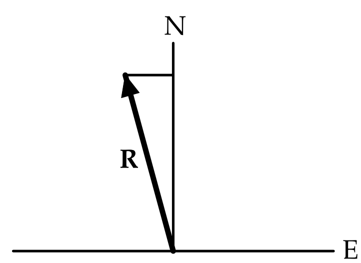


FIG. P3.24

The crow's velocity is

$$\mathbf{v} = \frac{\Delta\mathbf{x}}{\Delta t} = \frac{561\text{ m at }11.3°\text{ W of N}}{40\text{ s}}$$
$$= \boxed{14.0\text{ m/s at }11.3°\text{ west of north}}.$$

P3.25 $+x$ East, $+y$ North

$$\sum x = 250 + 125\cos 30° = 358 \text{ m}$$
$$\sum y = 75 + 125\sin 30° - 150 = -12.5 \text{ m}$$
$$d = \sqrt{\left(\sum x\right)^2 + \left(\sum y\right)^2} = \sqrt{(358)^2 + (-12.5)^2} = 358 \text{ m}$$
$$\tan\theta = \frac{\left(\sum y\right)}{\left(\sum x\right)} = -\frac{12.5}{358} = -0.0349$$
$$\theta = -2.00°$$
$$\boxed{\mathbf{d} = 358 \text{ m at } 2.00° \text{ S of E}}$$

P3.26 The east and north components of the displacement from Dallas (D) to Chicago (C) are the sums of the east and north components of the displacements from Dallas to Atlanta (A) and from Atlanta to Chicago. In equation form:

$$d_{\text{DC east}} = d_{\text{DA east}} + d_{\text{AC east}} = 730\cos 5.00° - 560\sin 21.0° = 527 \text{ miles}.$$
$$d_{\text{DC north}} = d_{\text{DA north}} + d_{\text{AC north}} = 730\sin 5.00° + 560\cos 21.0° = 586 \text{ miles}.$$

By the Pythagorean theorem, $d = \sqrt{(d_{\text{DC east}})^2 + (d_{\text{DC north}})^2} = 788 \text{ mi}$.

$$\text{Then } \tan\theta = \frac{d_{\text{DC north}}}{d_{\text{DC east}}} = 1.11 \text{ and } \theta = 48.0°.$$

Thus, Chicago is $\boxed{788 \text{ miles at } 48.0° \text{ northeast of Dallas}}$.

P3.27 (a) See figure to the right.

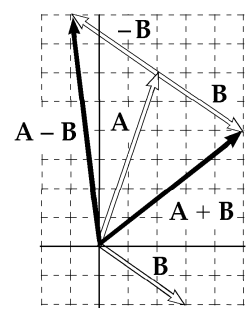


FIG. P3.27

(b) $\mathbf{C} = \mathbf{A} + \mathbf{B} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} + 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} = \boxed{5.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}}$

$$\mathbf{C} = \sqrt{25.0 + 16.0} \text{ at } \tan^{-1}\left(\frac{4}{5}\right) = \boxed{6.40 \text{ at } 38.7°}$$

$$\mathbf{D} = \mathbf{A} - \mathbf{B} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}} = \boxed{-1.00\hat{\mathbf{i}} + 8.00\hat{\mathbf{j}}}$$

$$\mathbf{D} = \sqrt{(-1.00)^2 + (8.00)^2} \text{ at } \tan^{-1}\left(\frac{8.00}{-1.00}\right)$$

$$\mathbf{D} = 8.06 \text{ at } (180° - 82.9°) = \boxed{8.06 \text{ at } 97.2°}$$

P3.28 $d = \sqrt{(x_1 + x_2 + x_3)^2 + (y_1 + y_2 + y_3)^2}$

$$= \sqrt{(3.00 - 5.00 + 6.00)^2 + (2.00 + 3.00 + 1.00)^2} = \sqrt{52.0} = \boxed{7.21 \text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{6.00}{4.00}\right) = \boxed{56.3°}$$

P3.29 We have $\mathbf{B} = \mathbf{R} - \mathbf{A}$:

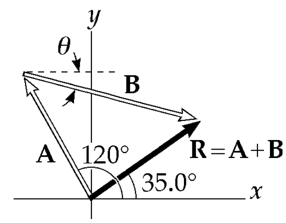


FIG. P3.29

$$A_x = 150\cos 120° = -75.0 \text{ cm}$$
$$A_y = 150\sin 120° = 130 \text{ cm}$$
$$R_x = 140\cos 35.0° = 115 \text{ cm}$$
$$R_y = 140\sin 35.0° = 80.3 \text{ cm}$$

Therefore,

$$\mathbf{B} = [115 - (-75)]\hat{\mathbf{i}} + [80.3 - 130]\hat{\mathbf{j}} = (190\hat{\mathbf{i}} - 49.7\hat{\mathbf{j}}) \text{ cm}$$
$$|\mathbf{B}| = \sqrt{190^2 + 49.7^2} = \boxed{196 \text{ cm}}$$
$$\theta = \tan^{-1}\left(-\frac{49.7}{190}\right) = \boxed{-14.7°}.$$

P3.30 $\mathbf{A} = -8.70\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}$ and $\mathbf{B} = 13.2\hat{\mathbf{i}} - 6.60\hat{\mathbf{j}}$

$\mathbf{A} - \mathbf{B} + 3\mathbf{C} = 0$:

$$3\mathbf{C} = \mathbf{B} - \mathbf{A} = 21.9\hat{\mathbf{i}} - 21.6\hat{\mathbf{j}}$$
$$\mathbf{C} = 7.30\hat{\mathbf{i}} - 7.20\hat{\mathbf{j}}$$

or

$$C_x = \boxed{7.30 \text{ cm}};\ C_y = \boxed{-7.20 \text{ cm}}$$

P3.31 (a) $(\mathbf{A} + \mathbf{B}) = (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) + (-\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) = \boxed{2\hat{\mathbf{i}} - 6\hat{\mathbf{j}}}$

(b) $(\mathbf{A} - \mathbf{B}) = (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) - (-\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) = \boxed{4\hat{\mathbf{i}} + 2\hat{\mathbf{j}}}$

(c) $|\mathbf{A} + \mathbf{B}| = \sqrt{2^2 + 6^2} = \boxed{6.32}$

(d) $|\mathbf{A} - \mathbf{B}| = \sqrt{4^2 + 2^2} = \boxed{4.47}$

(e) $\theta_{|\mathbf{A}+\mathbf{B}|} = \tan^{-1}\left(-\frac{6}{2}\right) = -71.6° = \boxed{288°}$

$\theta_{|\mathbf{A}-\mathbf{B}|} = \tan^{-1}\left(\frac{2}{4}\right) = \boxed{26.6°}$

P3.32 (a) $\mathbf{D} = \mathbf{A} + \mathbf{B} + \mathbf{C} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$

$|\mathbf{D}| = \sqrt{2^2 + 4^2} = \boxed{4.47 \text{ m at } \theta = 63.4°}$

(b) $\mathbf{E} = -\mathbf{A} - \mathbf{B} + \mathbf{C} = -6\hat{\mathbf{i}} + 6\hat{\mathbf{j}}$

$|\mathbf{E}| = \sqrt{6^2 + 6^2} = \boxed{8.49 \text{ m at } \theta = 135°}$

P3.33 $d_1 = \left(-3.50\hat{\mathbf{j}}\right)$ m

$d_2 = 8.20\cos 45.0°\,\hat{\mathbf{i}} + 8.20\sin 45.0°\,\hat{\mathbf{j}} = \left(5.80\hat{\mathbf{i}} + 5.80\hat{\mathbf{j}}\right)$ m

$d_3 = \left(-15.0\hat{\mathbf{i}}\right)$ m

$\mathbf{R} = d_1 + d_2 + d_3 = (-15.0 + 5.80)\hat{\mathbf{i}} + (5.80 - 3.50)\hat{\mathbf{j}} = \boxed{\left(-9.20\hat{\mathbf{i}} + 2.30\hat{\mathbf{j}}\right)\text{ m}}$

(or 9.20 m west and 2.30 m north)

The magnitude of the resultant displacement is

$$|\mathbf{R}| = \sqrt{R_x^2 + R_y^2} = \sqrt{(-9.20)^2 + (2.30)^2} = \boxed{9.48\text{ m}}.$$

The direction is $\theta = \arctan\left(\dfrac{2.30}{-9.20}\right) = \boxed{166°}$.

P3.34 Refer to the sketch

$$\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} = -10.0\hat{\mathbf{i}} - 15.0\hat{\mathbf{j}} + 50.0\hat{\mathbf{i}}$$
$$= 40.0\hat{\mathbf{i}} - 15.0\hat{\mathbf{j}}$$
$$|\mathbf{R}| = \left[(40.0)^2 + (-15.0)^2\right]^{1/2} = \boxed{42.7\text{ yards}}$$

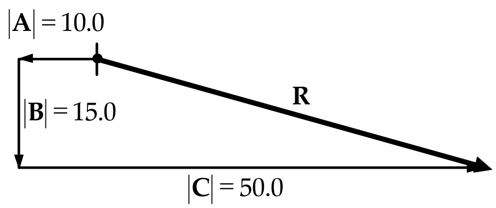


FIG. P3.34

P3.35 (a) $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$

$\mathbf{F} = 120\cos(60.0°)\hat{\mathbf{i}} + 120\sin(60.0°)\hat{\mathbf{j}} - 80.0\cos(75.0°)\hat{\mathbf{i}} + 80.0\sin(75.0°)\hat{\mathbf{j}}$

$\mathbf{F} = 60.0\hat{\mathbf{i}} + 104\hat{\mathbf{j}} - 20.7\hat{\mathbf{i}} + 77.3\hat{\mathbf{j}} = \left(39.3\hat{\mathbf{i}} + 181\hat{\mathbf{j}}\right)$ N

$|\mathbf{F}| = \sqrt{39.3^2 + 181^2} = \boxed{185\text{ N}}$

$\theta = \tan^{-1}\left(\dfrac{181}{39.3}\right) = \boxed{77.8°}$

(b) $\mathbf{F}_3 = -\mathbf{F} = \boxed{\left(-39.3\hat{\mathbf{i}} - 181\hat{\mathbf{j}}\right)\text{ N}}$

P3.36

East x	West y
0 m	4.00 m
1.41	1.41
–0.500	–0.866
+0.914	4.55

$|\mathbf{R}| = \sqrt{|x|^2 + |y|^2} = \boxed{4.64\text{ m at }78.6°\text{ N of E}}$

P3.37 $\mathbf{A} = 3.00\text{ m},\ \theta_A = 30.0°$ $\qquad$ $\mathbf{B} = 3.00\text{ m},\ \theta_B = 90.0°$

$A_x = A\cos\theta_A = 3.00\cos 30.0° = 2.60\text{ m}$ $\qquad$ $A_y = A\sin\theta_A = 3.00\sin 30.0° = 1.50\text{ m}$

$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = \left(2.60\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}\right)\text{ m}$

$B_x = 0,\ B_y = 3.00\text{ m}$ so $\mathbf{B} = 3.00\hat{\mathbf{j}}\text{ m}$

$\mathbf{A} + \mathbf{B} = \left(2.60\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}\right) + 3.00\hat{\mathbf{j}} = \boxed{\left(2.60\hat{\mathbf{i}} + 4.50\hat{\mathbf{j}}\right)\text{ m}}$

P3.38 Let the positive x-direction be eastward, the positive y-direction be vertically upward, and the positive z-direction be southward. The total displacement is then

$$\mathbf{d} = \left(4.80\hat{\mathbf{i}} + 4.80\hat{\mathbf{j}}\right)\text{ cm} + \left(3.70\hat{\mathbf{j}} - 3.70\hat{\mathbf{k}}\right)\text{ cm} = \left(4.80\hat{\mathbf{i}} + 8.50\hat{\mathbf{j}} - 3.70\hat{\mathbf{k}}\right)\text{ cm}.$$

(a) The magnitude is $d = \sqrt{(4.80)^2 + (8.50)^2 + (-3.70)^2}\text{ cm} = \boxed{10.4\text{ cm}}$.

(b) Its angle with the y-axis follows from $\cos\theta = \dfrac{8.50}{10.4}$, giving $\boxed{\theta = 35.5°}$.

P3.39 $\mathbf{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}} = 4.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} + 3.00\hat{\mathbf{k}}$

$|\mathbf{B}| = \sqrt{4.00^2 + 6.00^2 + 3.00^2} = \boxed{7.81}$

$\alpha = \cos^{-1}\left(\dfrac{4.00}{7.81}\right) = \boxed{59.2°}$

$\beta = \cos^{-1}\left(\dfrac{6.00}{7.81}\right) = \boxed{39.8°}$

$\gamma = \cos^{-1}\left(\dfrac{3.00}{7.81}\right) = \boxed{67.4°}$

P3.40 The y coordinate of the airplane is constant and equal to $7.60 \times 10^3\text{ m}$ whereas the x coordinate is given by $x = v_i t$ where v_i is the constant speed in the horizontal direction.

At $t = 30.0\text{ s}$ we have $x = 8.04 \times 10^3$, so $v_i = 268\text{ m/s}$. The position vector as a function of time is

$$\mathbf{P} = (268\text{ m/s})t\hat{\mathbf{i}} + \left(7.60 \times 10^3\text{ m}\right)\hat{\mathbf{j}}.$$

At $t = 45.0\text{ s}$, $\mathbf{P} = \left[1.21 \times 10^4\hat{\mathbf{i}} + 7.60 \times 10^3\hat{\mathbf{j}}\right]\text{ m}$. The magnitude is

$$\mathbf{P} = \sqrt{\left(1.21 \times 10^4\right)^2 + \left(7.60 \times 10^3\right)^2}\text{ m} = \boxed{1.43 \times 10^4\text{ m}}$$

and the direction is

$$\theta = \arctan\left(\frac{7.60 \times 10^3}{1.21 \times 10^4}\right) = \boxed{32.2°\text{ above the horizontal}}.$$

P3.41 (a) $\mathbf{A} = \boxed{8.00\hat{\mathbf{i}} + 12.0\hat{\mathbf{j}} - 4.00\hat{\mathbf{k}}}$

(b) $\mathbf{B} = \dfrac{\mathbf{A}}{4} = \boxed{2.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}} - 1.00\hat{\mathbf{k}}}$

(c) $\mathbf{C} = -3\mathbf{A} = \boxed{-24.0\hat{\mathbf{i}} - 36.0\hat{\mathbf{j}} + 12.0\hat{\mathbf{k}}}$

P3.42 $\mathbf{R} = 75.0\cos 240°\,\hat{\mathbf{i}} + 75.0\sin 240°\,\hat{\mathbf{j}} + 125\cos 135°\,\hat{\mathbf{i}} + 125\sin 135°\,\hat{\mathbf{j}} + 100\cos 160°\,\hat{\mathbf{i}} + 100\sin 160°\,\hat{\mathbf{j}}$

$\mathbf{R} = -37.5\hat{\mathbf{i}} - 65.0\hat{\mathbf{j}} - 88.4\hat{\mathbf{i}} + 88.4\hat{\mathbf{j}} - 94.0\hat{\mathbf{i}} + 34.2\hat{\mathbf{j}}$

$\mathbf{R} = \boxed{-220\hat{\mathbf{i}} + 57.6\hat{\mathbf{j}}}$

$\mathbf{R} = \sqrt{(-220)^2 + 57.6^2}$ at $\arctan\left(\dfrac{57.6}{220}\right)$ above the $-x$-axis

$\mathbf{R} = \boxed{227 \text{ paces at } 165°}$

P3.43 (a) $\mathbf{C} = \mathbf{A} + \mathbf{B} = \boxed{\left(5.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{k}}\right)\text{ m}}$

$|\mathbf{C}| = \sqrt{(5.00)^2 + (1.00)^2 + (3.00)^2}\text{ m} = \boxed{5.92\text{ m}}$

(b) $\mathbf{D} = 2\mathbf{A} - \mathbf{B} = \boxed{\left(4.00\hat{\mathbf{i}} - 11.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{k}}\right)\text{ m}}$

$|\mathbf{D}| = \sqrt{(4.00)^2 + (11.0)^2 + (15.0)^2}\text{ m} = \boxed{19.0\text{ m}}$

P3.44 The position vector from radar station to ship is

$$\mathbf{S} = \left(17.3\sin 136°\,\hat{\mathbf{i}} + 17.3\cos 136°\,\hat{\mathbf{j}}\right)\text{ km} = \left(12.0\hat{\mathbf{i}} - 12.4\hat{\mathbf{j}}\right)\text{ km}.$$

From station to plane, the position vector is

$$\mathbf{P} = \left(19.6\sin 153°\,\hat{\mathbf{i}} + 19.6\cos 153°\,\hat{\mathbf{j}} + 2.20\hat{\mathbf{k}}\right)\text{ km},$$

or

$$\mathbf{P} = \left(8.90\hat{\mathbf{i}} - 17.5\hat{\mathbf{j}} + 2.20\hat{\mathbf{k}}\right)\text{ km}.$$

(a) To fly to the ship, the plane must undergo displacement

$$\mathbf{D} = \mathbf{S} - \mathbf{P} = \boxed{\left(3.12\hat{\mathbf{i}} + 5.02\hat{\mathbf{j}} - 2.20\hat{\mathbf{k}}\right)\text{ km}}.$$

(b) The distance the plane must travel is

$$D = |\mathbf{D}| = \sqrt{(3.12)^2 + (5.02)^2 + (2.20)^2}\text{ km} = \boxed{6.31\text{ km}}.$$

P3.45 The hurricane's first displacement is $\left(\frac{41.0\text{ km}}{\text{h}}\right)(3.00\text{ h})$ at $60.0°$ N of W, and its second displacement is $\left(\frac{25.0\text{ km}}{\text{h}}\right)(1.50\text{ h})$ due North. With $\hat{\mathbf{i}}$ representing east and $\hat{\mathbf{j}}$ representing north, its total displacement is:

$$\left(41.0\frac{\text{km}}{\text{h}}\cos 60.0°\right)(3.00\text{ h})\left(-\hat{\mathbf{i}}\right)+\left(41.0\frac{\text{km}}{\text{h}}\sin 60.0°\right)(3.00\text{ h})\hat{\mathbf{j}}+\left(25.0\frac{\text{km}}{\text{h}}\right)(1.50\text{ h})\hat{\mathbf{j}}=61.5\text{ km}\left(-\hat{\mathbf{i}}\right)$$
$$+144\text{ km}\,\hat{\mathbf{j}}$$

with magnitude $\sqrt{(61.5\text{ km})^2+(144\text{ km})^2}=\boxed{157\text{ km}}$.

P3.46 (a) $\mathbf{E}=(17.0\text{ cm})\cos 27.0°\,\hat{\mathbf{i}}+(17.0\text{ cm})\sin 27.0°\,\hat{\mathbf{j}}$

$\mathbf{E}=\boxed{\left(15.1\hat{\mathbf{i}}+7.72\hat{\mathbf{j}}\right)\text{ cm}}$

(b) $\mathbf{F}=-(17.0\text{ cm})\sin 27.0°\,\hat{\mathbf{i}}+(17.0\text{ cm})\cos 27.0°\,\hat{\mathbf{j}}$

$\mathbf{F}=\boxed{\left(-7.72\hat{\mathbf{i}}+15.1\hat{\mathbf{j}}\right)\text{ cm}}$

(c) $\mathbf{G}=+(17.0\text{ cm})\sin 27.0°\,\hat{\mathbf{i}}+(17.0\text{ cm})\cos 27.0°\,\hat{\mathbf{j}}$

$\mathbf{G}=\boxed{\left(+7.72\hat{\mathbf{i}}+15.1\hat{\mathbf{j}}\right)\text{ cm}}$

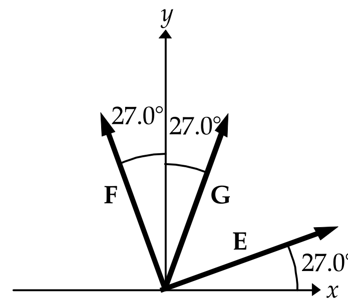


FIG. P3.46

P3.47 $A_x=-3.00$, $A_y=2.00$

(a) $\mathbf{A}=A_x\hat{\mathbf{i}}+A_y\hat{\mathbf{j}}=\boxed{-3.00\hat{\mathbf{i}}+2.00\hat{\mathbf{j}}}$

(b) $|\mathbf{A}|=\sqrt{A_x^2+A_y^2}=\sqrt{(-3.00)^2+(2.00)^2}=\boxed{3.61}$

$\tan\theta=\frac{A_y}{A_x}=\frac{2.00}{(-3.00)}=-0.667$, $\tan^{-1}(-0.667)=-33.7°$

θ is in the 2^{nd} quadrant, so $\theta=180°+(-33.7°)=\boxed{146°}$.

(c) $\mathbf{R}_x=0$, $\mathbf{R}_y=-4.00$, $\mathbf{R}=\mathbf{A}+\mathbf{B}$ thus $\mathbf{B}=\mathbf{R}-\mathbf{A}$ and

$B_x=R_x-A_x=0-(-3.00)=3.00$, $B_y=R_y-A_y=-4.00-2.00=-6.00$.

Therefore, $\mathbf{B}=\boxed{3.00\hat{\mathbf{i}}-6.00\hat{\mathbf{j}}}$.

P3.48 Let $+x =$ East, $+y =$ North,

x	y
300	0
–175	303
0	150
125	453

(a) $\theta = \tan^{-1}\frac{y}{x} = \boxed{74.6° \text{ N of E}}$

(b) $|\mathbf{R}| = \sqrt{x^2 + y^2} = \boxed{470 \text{ km}}$

P3.49 (a) $R_x = 40.0\cos 45.0° + 30.0\cos 45.0° = 49.5$

$R_y = 40.0\sin 45.0° - 30.0\sin 45.0° + 20.0 = 27.1$

$\mathbf{R} = \boxed{49.5\hat{\mathbf{i}} + 27.1\hat{\mathbf{j}}}$

(b) $|\mathbf{R}| = \sqrt{(49.5)^2 + (27.1)^2} = \boxed{56.4}$

$\theta = \tan^{-1}\left(\frac{27.1}{49.5}\right) = \boxed{28.7°}$

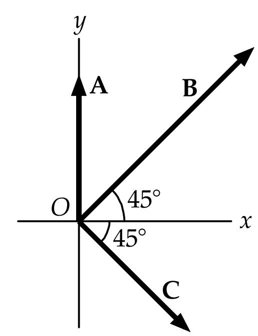


FIG. P3.49

P3.50 Taking components along $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, we get two equations:

$$6.00a - 8.00b + 26.0 = 0$$

and

$$-8.00a + 3.00b + 19.0 = 0.$$

Solving simultaneously,

$$\boxed{a = 5.00,\ b = 7.00}.$$

Therefore,

$$5.00\mathbf{A} + 7.00\mathbf{B} + \mathbf{C} = 0.$$

Additional Problems

P3.51 Let θ represent the angle between the directions of **A** and **B**. Since **A** and **B** have the same magnitudes, **A**, **B**, and $\mathbf{R} = \mathbf{A} + \mathbf{B}$ form an isosceles triangle in which the angles are $180° - \theta$, $\frac{\theta}{2}$, and $\frac{\theta}{2}$. The magnitude of **R** is then $R = 2A\cos\left(\frac{\theta}{2}\right)$. [*Hint:* apply the law of cosines to the isosceles triangle and use the fact that $B = A$.]

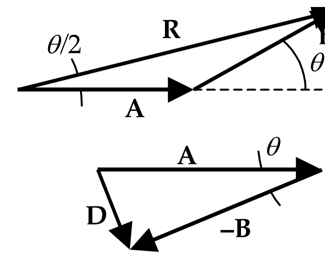


FIG. P3.51

Again, **A**, –**B**, and $\mathbf{D} = \mathbf{A} - \mathbf{B}$ form an isosceles triangle with apex angle θ. Applying the law of cosines and the identity

$$(1 - \cos\theta) = 2\sin^2\left(\frac{\theta}{2}\right)$$

gives the magnitude of **D** as $D = 2A\sin\left(\frac{\theta}{2}\right)$.

The problem requires that $R = 100D$.

Thus, $2A\cos\left(\frac{\theta}{2}\right) = 200A\sin\left(\frac{\theta}{2}\right)$. This gives $\tan\left(\frac{\theta}{2}\right) = 0.010$ and $\boxed{\theta = 1.15°}$.

P3.52 Let θ represent the angle between the directions of **A** and **B**. Since **A** and **B** have the same magnitudes, **A**, **B**, and $\mathbf{R} = \mathbf{A} + \mathbf{B}$ form an isosceles triangle in which the angles are $180° - \theta$, $\frac{\theta}{2}$, and $\frac{\theta}{2}$. The magnitude of **R** is then $R = 2A\cos\left(\frac{\theta}{2}\right)$. [*Hint*: apply the law of cosines to the isosceles triangle and use the fact that $B = A$.]

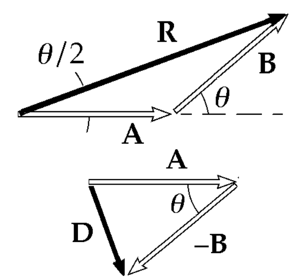


FIG. P3.52

Again, **A**, –**B**, and $\mathbf{D} = \mathbf{A} - \mathbf{B}$ form an isosceles triangle with apex angle θ. Applying the law of cosines and the identity

$$(1 - \cos\theta) = 2\sin^2\left(\frac{\theta}{2}\right)$$

gives the magnitude of **D** as $D = 2A\sin\left(\frac{\theta}{2}\right)$.

The problem requires that $R = nD$ or $\cos\left(\frac{\theta}{2}\right) = n\sin\left(\frac{\theta}{2}\right)$ giving

$\boxed{\theta = 2\tan^{-1}\left(\frac{1}{n}\right)}$.

P3.53 (a) $R_x = \boxed{2.00}$, $R_y = \boxed{1.00}$, $R_z = \boxed{3.00}$

(b) $|\mathbf{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{4.00 + 1.00 + 9.00} = \sqrt{14.0} = \boxed{3.74}$

(c)
$$\cos\theta_x = \frac{R_x}{|\mathbf{R}|} \Rightarrow \theta_x = \cos^{-1}\left(\frac{R_x}{|\mathbf{R}|}\right) = \boxed{57.7° \text{ from } +x}$$
$$\cos\theta_y = \frac{R_y}{|\mathbf{R}|} \Rightarrow \theta_y = \cos^{-1}\left(\frac{R_y}{|\mathbf{R}|}\right) = \boxed{74.5° \text{ from } +y}$$
$$\cos\theta_z = \frac{R_z}{|\mathbf{R}|} \Rightarrow \theta_z = \cos^{-1}\left(\frac{R_z}{|\mathbf{R}|}\right) = \boxed{36.7° \text{ from } +z}$$

***P3.54** Take the *x*-axis along the tail section of the snake. The displacement from tail to head is

$$240 \text{ m}\hat{\mathbf{i}} + (420 - 240) \text{ m} \cos(180° – 105°)\hat{\mathbf{i}} - 180 \text{ m} \sin 75° \,\hat{\mathbf{j}} = 287 \text{ m}\hat{\mathbf{i}} - 174 \text{ m}\hat{\mathbf{j}}.$$

Its magnitude is $\sqrt{(287)^2 + (174)^2}$ m $= 335$ m. From $v = \dfrac{\text{distance}}{\Delta t}$, the time for each child's run is

$$\text{Inge: } \Delta t = \frac{\text{distance}}{v} = \frac{335 \text{ m(h)(1 km)(3 600 s)}}{(12 \text{ km})(1\,000 \text{ m})(1 \text{ h})} = 101 \text{ s}$$
$$\text{Olaf: } \Delta t = \frac{420 \text{ m}\cdot\text{s}}{3.33 \text{ m}} = 126 \text{ s}.$$

Inge wins by $126 - 101 = \boxed{25.4 \text{ s}}$.

***P3.55** The position vector from the ground under the controller of the first airplane is

$$\mathbf{r}_1 = (19.2 \text{ km})(\cos 25°)\hat{\mathbf{i}} + (19.2 \text{ km})(\sin 25°)\hat{\mathbf{j}} + (0.8 \text{ km})\hat{\mathbf{k}}$$
$$= \left(17.4\hat{\mathbf{i}} + 8.11\hat{\mathbf{j}} + 0.8\hat{\mathbf{k}}\right) \text{ km}.$$

The second is at

$$\mathbf{r}_2 = (17.6 \text{ km})(\cos 20°)\hat{\mathbf{i}} + (17.6 \text{ km})(\sin 20°)\hat{\mathbf{j}} + (1.1 \text{ km})\hat{\mathbf{k}}$$
$$= \left(16.5\hat{\mathbf{i}} + 6.02\hat{\mathbf{j}} + 1.1\hat{\mathbf{k}}\right) \text{ km}.$$

Now the displacement from the first plane to the second is

$$\mathbf{r}_2 - \mathbf{r}_1 = \left(-0.863\hat{\mathbf{i}} - 2.09\hat{\mathbf{j}} + 0.3\hat{\mathbf{k}}\right) \text{ km}$$

with magnitude

$$\sqrt{(0.863)^2 + (2.09)^2 + (0.3)^2} = \boxed{2.29 \text{ km}}.$$

***P3.56** Let A represent the distance from island 2 to island 3. The displacement is $\mathbf{A} = A$ at $159°$. Represent the displacement from 3 to 1 as $\mathbf{B} = B$ at $298°$. We have 4.76 km at $37°$ $+\mathbf{A} + \mathbf{B} = 0$.

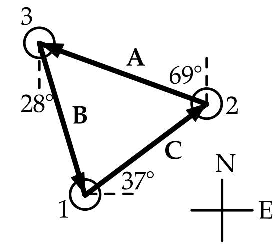


FIG. P3.56

For x-components

$$(4.76\text{ km})\cos 37° + A\cos 159° + B\cos 298° = 0$$
$$3.80\text{ km} - 0.934A + 0.469B = 0$$
$$B = -8.10\text{ km} + 1.99A$$

For y-components,

$$(4.76\text{ km})\sin 37° + A\sin 159° + B\sin 298° = 0$$
$$2.86\text{ km} + 0.358A - 0.883B = 0$$

(a) We solve by eliminating B by substitution:

$$2.86\text{ km} + 0.358A - 0.883(-8.10\text{ km} + 1.99A) = 0$$
$$2.86\text{ km} + 0.358A + 7.15\text{ km} - 1.76A = 0$$
$$10.0\text{ km} = 1.40A$$
$$A = \boxed{7.17\text{ km}}$$

(b) $B = -8.10\text{ km} + 1.99(7.17\text{ km}) = \boxed{6.15\text{ km}}$

***P3.57** (a) We first express the corner's position vectors as sets of components

$$\mathbf{A} = (10\text{ m})\cos 50°\,\hat{\mathbf{i}} + (10\text{ m})\sin 50°\,\hat{\mathbf{j}} = 6.43\text{ m}\hat{\mathbf{i}} + 7.66\text{ m}\hat{\mathbf{j}}$$
$$\mathbf{B} = (12\text{ m})\cos 30°\,\hat{\mathbf{i}} + (12\text{ m})\sin 30°\,\hat{\mathbf{j}} = 10.4\text{ m}\hat{\mathbf{i}} + 6.00\text{ m}\hat{\mathbf{j}}.$$

The horizontal width of the rectangle is

$$10.4\text{ m} - 6.43\text{ m} = 3.96\text{ m}.$$

Its vertical height is

$$7.66\text{ m} - 6.00\text{ m} = 1.66\text{ m}.$$

Its perimeter is

$$2(3.96 + 1.66)\text{ m} = \boxed{11.2\text{ m}}.$$

(b) The position vector of the distant corner is $B_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = 10.4\text{ m}\hat{\mathbf{i}} + 7.66\text{ m}\hat{\mathbf{j}} = \sqrt{10.4^2 + 7.66^2}$ m at $\tan^{-1}\dfrac{7.66\text{ m}}{10.4\text{ m}} = \boxed{12.9\text{ m at } 36.4°}$.

P3.58 Choose the $+x$-axis in the direction of the first force. The total force, in newtons, is then

$$12.0\hat{\mathbf{i}}+31.0\hat{\mathbf{j}}-8.40\hat{\mathbf{i}}-24.0\hat{\mathbf{j}}=\boxed{(3.60\hat{\mathbf{i}})+(7.00\hat{\mathbf{j}})\text{ N}}.$$

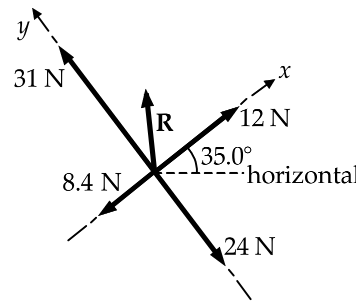


FIG. P3.58

The magnitude of the total force is

$$\sqrt{(3.60)^2+(7.00)^2}\text{ N}=\boxed{7.87\text{ N}}$$

and the angle it makes with our $+x$-axis is given by $\tan\theta=\frac{(7.00)}{(3.60)}$, $\theta=62.8°$. Thus, its angle counterclockwise from the horizontal is $35.0°+62.8°=\boxed{97.8°}$.

P3.59

$$\mathbf{d}_1=100\hat{\mathbf{i}}$$
$$\mathbf{d}_2=-300\hat{\mathbf{j}}$$
$$\mathbf{d}_3=-150\cos(30.0°)\hat{\mathbf{i}}-150\sin(30.0°)\hat{\mathbf{j}}=-130\hat{\mathbf{i}}-75.0\hat{\mathbf{j}}$$
$$\mathbf{d}_4=-200\cos(60.0°)\hat{\mathbf{i}}+200\sin(60.0°)\hat{\mathbf{j}}=-100\hat{\mathbf{i}}+173\hat{\mathbf{j}}$$
$$\mathbf{R}=\mathbf{d}_1+\mathbf{d}_2+\mathbf{d}_3+\mathbf{d}_4=\boxed{(-130\hat{\mathbf{i}}-202\hat{\mathbf{j}})\text{ m}}$$
$$|\mathbf{R}|=\sqrt{(-130)^2+(-202)^2}=\boxed{240\text{ m}}$$
$$\phi=\tan^{-1}\left(\frac{202}{130}\right)=57.2°$$
$$\theta=180+\phi=\boxed{237°}$$

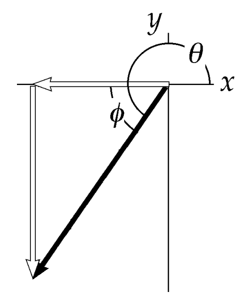


FIG. P3.59

P3.60

$$\frac{d\mathbf{r}}{dt}=\frac{d(4\hat{\mathbf{i}}+3\hat{\mathbf{j}}-2t\hat{\mathbf{j}})}{dt}=0+0-2\hat{\mathbf{j}}=\boxed{-(2.00\text{ m/s})\hat{\mathbf{j}}}$$

The position vector at $t=0$ is $4\hat{\mathbf{i}}+3\hat{\mathbf{j}}$. At $t=1$ s, the position is $4\hat{\mathbf{i}}+1\hat{\mathbf{j}}$, and so on. The object is moving straight downward at 2 m/s, so

$$\frac{d\mathbf{r}}{dt}\text{ represents }\boxed{\text{its velocity vector}}.$$

P3.61

$$\mathbf{v}=v_x\hat{\mathbf{i}}+v_y\hat{\mathbf{j}}=(300+100\cos30.0°)\hat{\mathbf{i}}+(100\sin30.0°)\hat{\mathbf{j}}$$
$$\mathbf{v}=(387\hat{\mathbf{i}}+50.0\hat{\mathbf{j}})\text{ mi/h}$$
$$|\mathbf{v}|=\boxed{390\text{ mi/h at }7.37°\text{ N of E}}$$

P3.62 (a) You start at point A: $\mathbf{r}_1 = \mathbf{r}_A = \left(30.0\hat{\mathbf{i}} - 20.0\hat{\mathbf{j}}\right)$ m.

The displacement to B is

$$\mathbf{r}_B - \mathbf{r}_A = 60.0\hat{\mathbf{i}} + 80.0\hat{\mathbf{j}} - 30.0\hat{\mathbf{i}} + 20.0\hat{\mathbf{j}} = 30.0\hat{\mathbf{i}} + 100\hat{\mathbf{j}}.$$

You cover half of this, $\left(15.0\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}}\right)$ to move to $\mathbf{r}_2 = 30.0\hat{\mathbf{i}} - 20.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}} = 45.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}}$.

Now the displacement from your current position to C is

$$\mathbf{r}_C - \mathbf{r}_2 = -10.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}} - 45.0\hat{\mathbf{i}} - 30.0\hat{\mathbf{j}} = -55.0\hat{\mathbf{i}} - 40.0\hat{\mathbf{j}}.$$

You cover one-third, moving to

$$\mathbf{r}_3 = \mathbf{r}_2 + \Delta\mathbf{r}_{23} = 45.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}} + \frac{1}{3}\left(-55.0\hat{\mathbf{i}} - 40.0\hat{\mathbf{j}}\right) = 26.7\hat{\mathbf{i}} + 16.7\hat{\mathbf{j}}.$$

The displacement from where you are to D is

$$\mathbf{r}_D - \mathbf{r}_3 = 40.0\hat{\mathbf{i}} - 30.0\hat{\mathbf{j}} - 26.7\hat{\mathbf{i}} - 16.7\hat{\mathbf{j}} = 13.3\hat{\mathbf{i}} - 46.7\hat{\mathbf{j}}.$$

You traverse one-quarter of it, moving to

$$\mathbf{r}_4 = \mathbf{r}_3 + \frac{1}{4}\left(\mathbf{r}_D - \mathbf{r}_3\right) = 26.7\hat{\mathbf{i}} + 16.7\hat{\mathbf{j}} + \frac{1}{4}\left(13.3\hat{\mathbf{i}} - 46.7\hat{\mathbf{j}}\right) = 30.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}.$$

The displacement from your new location to **E** is

$$\mathbf{r}_E - \mathbf{r}_4 = -70.0\hat{\mathbf{i}} + 60.0\hat{\mathbf{j}} - 30.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}} = -100\hat{\mathbf{i}} + 55.0\hat{\mathbf{j}}$$

of which you cover one-fifth the distance, $-20.0\hat{\mathbf{i}} + 11.0\hat{\mathbf{j}}$, moving to

$$\mathbf{r}_4 + \Delta\mathbf{r}_{45} = 30.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}} - 20.0\hat{\mathbf{i}} + 11.0\hat{\mathbf{j}} = 10.0\hat{\mathbf{i}} + 16.0\hat{\mathbf{j}}.$$

The treasure is at $\boxed{(10.0\text{ m, }16.0\text{ m})}$.

(b) Following the directions brings you to the average position of the trees. The steps we took numerically in part (a) bring you to

$$\mathbf{r}_A + \frac{1}{2}\left(\mathbf{r}_B - \mathbf{r}_A\right) = \left(\frac{\mathbf{r}_A + \mathbf{r}_B}{2}\right)$$

then to $\dfrac{(\mathbf{r}_A + \mathbf{r}_B)}{2} + \dfrac{\mathbf{r}_C - \frac{(\mathbf{r}_A + \mathbf{r}_B)}{2}}{3} = \dfrac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C}{3}$

then to $\dfrac{(\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C)}{3} + \dfrac{\mathbf{r}_D - \frac{(\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C)}{3}}{4} = \dfrac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D}{4}$

and last to $\dfrac{(\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D)}{4} + \dfrac{\mathbf{r}_E - \frac{(\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D)}{4}}{5} = \dfrac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D + \mathbf{r}_E}{5}$.

This center of mass of the tree distribution is the same location whatever order we take the trees in.

***P3.63** (a) Let T represent the force exerted by each child. The x-component of the resultant force is

$$T\cos 0 + T\cos 120° + T\cos 240° = T(1) + T(-0.5) + T(-0.5) = 0.$$

The y-component is

$$T\sin 0 + T\sin 120 + T\sin 240 = 0 + 0.866T - 0.866T = 0.$$

Thus,

$$\sum \mathbf{F} = 0.$$

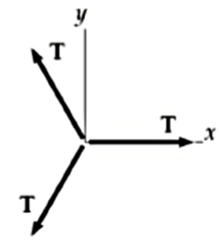


FIG. P3.63

(b) If the total force is not zero, it must point in some direction. When each child moves one space clockwise, the total must turn clockwise by that angle, $\frac{360°}{N}$. Since each child exerts the same force, the new situation is identical to the old and the net force on the tire must still point in the original direction. The contradiction indicates that we were wrong in supposing that the total force is not zero. The total force *must* be zero.

P3.64 (a) From the picture, $\mathbf{R}_1 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ and $|\mathbf{R}_1| = \sqrt{a^2 + b^2}$.

(b) $\mathbf{R}_2 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$; its magnitude is

$$\sqrt{|\mathbf{R}_1|^2 + c^2} = \sqrt{a^2 + b^2 + c^2}.$$

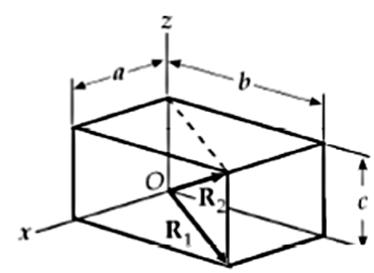


FIG. P3.64

P3.65 Since

$$\mathbf{A}+\mathbf{B}=6.00\hat{\mathbf{j}},$$

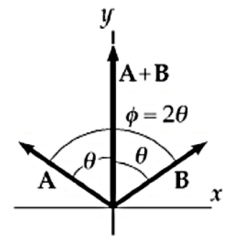


FIG. P3.65

we have

$$(A_x+B_x)\hat{\mathbf{i}}+(A_y+B_y)\hat{\mathbf{j}}=0\hat{\mathbf{i}}+6.00\hat{\mathbf{j}}$$

giving

$$A_x+B_x=0 \text{ or } A_x=-B_x \qquad [1]$$

and

$$A_y+B_y=6.00. \qquad [2]$$

Since both vectors have a magnitude of 5.00, we also have

$$A_x^2+A_y^2=B_x^2+B_y^2=5.00^2.$$

From $A_x=-B_x$, it is seen that

$$A_x^2=B_x^2.$$

Therefore, $A_x^2+A_y^2=B_x^2+B_y^2$ gives

$$A_y^2=B_y^2.$$

Then, $A_y=B_y$ and Eq. [2] gives

$$A_y=B_y=3.00.$$

Defining θ as the angle between either **A** or **B** and the y axis, it is seen that

$$\cos\theta=\frac{A_y}{A}=\frac{B_y}{B}=\frac{3.00}{5.00}=0.600 \text{ and } \theta=53.1°.$$

The angle between **A** and **B** is then $\boxed{\phi=2\theta=106°}$.

***P3.66** Let θ represent the angle the x-axis makes with the horizontal. Since angles are equal if their sides are perpendicular right side to right side and left side to left side, θ is also the angle between the weight and our y axis. The x-components of the forces must add to zero:

$$-0.150\text{ N}\sin\theta + 0.127\text{ N} = 0\,.$$

(b) $\theta = \boxed{57.9°}$

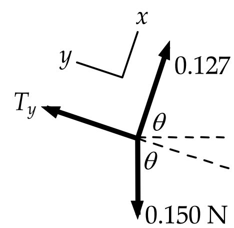


FIG. P3.66

(a) The y-components for the forces must add to zero:

$$+T_y - (0.150\text{ N})\cos 57.9° = 0\,,\; T_y = \boxed{0.079\,8\text{ N}}\,.$$

(c) The angle between the y axis and the horizontal is $90.0° - 57.9° = \boxed{32.1°}$.

P3.67 The displacement of point P is invariant under rotation of the coordinates. Therefore, $r = r'$ and $r^2 = (r')^2$ or, $x^2 + y^2 = (x')^2 + (y')^2$. Also, from the figure, $\beta = \theta - \alpha$

$$\therefore \tan^{-1}\left(\frac{y'}{x'}\right) = \tan^{-1}\left(\frac{y}{x}\right) - \alpha$$

$$\frac{y'}{x'} = \frac{\left(\frac{y}{x}\right) - \tan\alpha}{1 + \left(\frac{y}{x}\right)\tan\alpha}$$

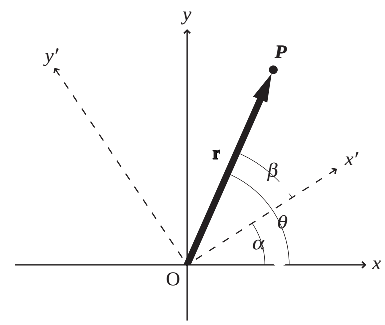


FIG. P3.67

Which we simplify by multiplying top and bottom by $x\cos\alpha$. Then,

$$x' = x\cos\alpha + y\sin\alpha,\; y' = -x\sin\alpha + y\cos\alpha.$$

ANSWERS TO EVEN PROBLEMS

P3.2 (a) (2.17 m, 1.25 m); (–1.90 m, 3.29 m); (b) 4.55 m

P3.4 (a) 8.60 m; (b) 4.47 m at –63.4°; 4.24 m at 135°

P3.6 (a) r at 180°–θ; (b) $2r$ at 180°+θ; (c) $3r$ at $-\theta$

P3.8 14 km at 65° north of east

P3.10 (a) 6.1 at 112°; (b) 14.8 at 22°

P3.12 9.5 N at 57°

P3.14 7.9 m at 4° north of west

P3.16 see the solution

P3.18 86.6 m and –50.0 m

P3.20 1.31 km north; 2.81 km east

P3.22 $-25.0\text{ m }\hat{\mathbf{i}} + 43.3\text{ m }\hat{\mathbf{j}}$

P3.24 14.0 m/s at 11.3° west of north

P3.26 788 mi at 48.0° north of east

P3.28 7.21 m at 56.3°

P3.30 $\mathbf{C} = 7.30\text{ cm }\hat{\mathbf{i}} - 7.20\text{ cm }\hat{\mathbf{j}}$

P3.32 (a) 4.47 m at 63.4°; (b) 8.49 m at 135°

P3.34 42.7 yards

P3.36 4.64 m at 78.6°

P3.38 (a) 10.4 cm; (b) 35.5°

P3.40 1.43×10^4 m at 32.2° above the horizontal

P3.42 $-220\hat{\mathbf{i}} + 57.6\hat{\mathbf{j}} = 227$ paces at 165°

P3.44 (a) $\left(3.12\hat{\mathbf{i}} + 5.02\hat{\mathbf{j}} - 2.20\hat{\mathbf{k}}\right)$ km; (b) 6.31 km

P3.46 (a) $\left(15.1\hat{\mathbf{i}} + 7.72\hat{\mathbf{j}}\right)$ cm;
(b) $\left(-7.72\hat{\mathbf{i}} + 15.1\hat{\mathbf{j}}\right)$ cm;
(c) $\left(+7.72\hat{\mathbf{i}} + 15.1\hat{\mathbf{j}}\right)$ cm

P3.48 (a) 74.6° north of east; (b) 470 km

P3.50 $a = 5.00$, $b = 7.00$

P3.52 $2\tan^{-1}\left(\frac{1}{n}\right)$

P3.54 25.4 s

P3.56 (a) 7.17 km; (b) 6.15 km

P3.58 7.87 N at 97.8° counterclockwise from a horizontal line to the right

P3.60 $(-2.00 \text{ m/s})\hat{\mathbf{j}}$; its velocity vector

P3.62 (a) (10.0 m, 16.0 m); (b) see the solution

P3.64 (a) $\mathbf{R}_1 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$; $|\mathbf{R}_1| = \sqrt{a^2 + b^2}$;
(b) $\mathbf{R}_2 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$; $|\mathbf{R}_2| = \sqrt{a^2 + b^2 + c^2}$

P3.66 (a) 0.079 8N; (b) 57.9°; (c) 32.1°