7

Energy and Energy Transfer

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q7.1 The force is perpendicular to every increment of displacement. Therefore, $\mathbf{F}\cdot\Delta\mathbf{r}=0$.

Q7.2 (a) Positive work is done by the chicken on the dirt.

(b) No work is done, although it may seem like there is.

(c) Positive work is done on the bucket.

(d) Negative work is done on the bucket.

(e) Negative work is done on the person's torso.

Q7.3 Yes. Force times distance over which the toe is in contact with the ball. No, he is no longer applying a force. Yes, both air friction and gravity do work.

Q7.4 Force of tension on a ball rotating on the end of a string. Normal force and gravitational force on an object at rest or moving across a level floor.

Q7.5 (a) Tension (b) Air resistance

(c) Positive in increasing velocity on the downswing.
Negative in decreasing velocity on the upswing.

Q7.6 No. The vectors might be in the third and fourth quadrants, but if the angle between them is less than 90° their dot product is positive.

Q7.7 The scalar product of two vectors is positive if the angle between them is between 0 and 90°. The scalar product is negative when $90°<\theta<180°$.

Q7.8 If the coils of the spring are initially in contact with one another, as the load increases from zero, the graph would be an upwardly curved arc. After the load increases sufficiently, the graph will be linear, described by Hooke's Law. This linear region will be quite large compared to the first region. The graph will then be a downward curved arc as the coiled spring becomes a completely straight wire. As the load increases with a straight wire, the graph will become a straight line again, with a significantly smaller slope. Eventually, the wire would break.

Q7.9 $k'=2k$. To stretch the smaller piece one meter, each coil would have to stretch twice as much as one coil in the original long spring, since there would be half as many coils. Assuming that the spring is ideal, twice the stretch requires twice the force.

Q7.10 Kinetic energy is always positive. Mass and squared speed are both positive. A moving object can always do positive work in striking another object and causing it to move along the same direction of motion.

Q7.11 Work is only done in accelerating the ball from rest. The work is done over the effective length of the pitcher's arm—the distance his hand moves through windup and until release.

Q7.12 Kinetic energy is proportional to mass. The first bullet has twice as much kinetic energy.

Q7.13 The longer barrel will have the higher muzzle speed. Since the accelerating force acts over a longer distance, the change in kinetic energy will be larger.

Q7.14 (a) Kinetic energy is proportional to squared speed. Doubling the speed makes an object's kinetic energy four times larger.

(b) If the total work on an object is zero in some process, its speed must be the same at the final point as it was at the initial point.

Q7.15 The larger engine is unnecessary. Consider a 30 minute commute. If you travel the same speed in each car, it will take the same amount of time, expending the same amount of energy. The extra power available from the larger engine isn't used.

Q7.16 If the instantaneous power output by some agent changes continuously, its average power in a process must be equal to its instantaneous power at least one instant. If its power output is constant, its instantaneous power is always equal to its average power.

Q7.17 It decreases, as the force required to lift the car decreases.

Q7.18 As you ride an express subway train, a backpack at your feet has no kinetic energy as measured by you since, according to you, the backpack is not moving. In the frame of reference of someone on the side of the tracks as the train rolls by, the backpack is moving and has mass, and thus has kinetic energy.

Q7.19 The rock increases in speed. The farther it has fallen, the more force it might exert on the sand at the bottom; but it might instead make a deeper crater with an equal-size average force. The farther it falls, the more work it will do in stopping. Its kinetic energy is increasing due to the work that the gravitational force does on it.

Q7.20 The normal force does no work because the angle between the normal force and the direction of motion is usually 90°. Static friction usually does no work because there is no distance through which the force is applied.

Q7.21 An argument for: As a glider moves along an airtrack, the only force that the track applies on the glider is the normal force. Since the angle between the direction of motion and the normal force is 90°, the work done must be zero, even if the track is not level.
Against: An airtrack has bumpers. When a glider bounces from the bumper at the end of the airtrack, it loses a bit of energy, as evidenced by a decreased speed. The airtrack does negative work.

Q7.22 Gaspard de Coriolis first stated the work-kinetic energy theorem. Jean Victor Poncelet, an engineer who invaded Russia with Napoleon, is most responsible for demonstrating its wide practical applicability, in his 1829 book *Industrial Mechanics*. Their work came remarkably late compared to the elucidation of momentum conservation in collisions by Descartes and to Newton's *Mathematical Principles of the Philosophy of Nature*, both in the 1600's.

SOLUTIONS TO PROBLEMS

Section 7.1 **Systems and Environments**

Section 7.2 **Work Done by a Constant Force**

P7.1 (a) $W = F\Delta r\cos\theta = (16.0\text{ N})(2.20\text{ m})\cos 25.0° = \boxed{31.9\text{ J}}$

(b), (c) The normal force and the weight are both at 90° to the displacement in any time interval. Both do $\boxed{0}$ work.

(d) $\sum W = 31.9\text{ J} + 0 + 0 = \boxed{31.9\text{ J}}$

P7.2 The component of force along the direction of motion is

$$F\cos\theta = (35.0\text{ N})\cos 25.0° = 31.7\text{ N}.$$

The work done by this force is

$$W = (F\cos\theta)\Delta r = (31.7\text{ N})(50.0\text{ m}) = \boxed{1.59\times10^3\text{ J}}.$$

P7.3 **Method One.**

Let ϕ represent the instantaneous angle the rope makes with the vertical as it is swinging up from $\phi_i = 0$ to $\phi_f = 60°$. In an incremental bit of motion from angle ϕ to $\phi + d\phi$, the definition of radian measure implies that $\Delta r = (12\text{ m})d\phi$. The angle θ between the incremental displacement and the force of gravity is $\theta = 90° + \phi$. Then $\cos\theta = \cos(90° + \phi) = -\sin\phi$.
The work done by the gravitational force on Batman is

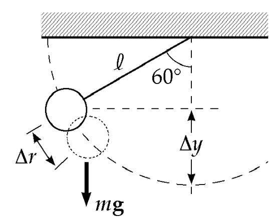

FIG. P7.3

$$W = \int_i^f F\cos\theta\,dr = \int_{\phi=0}^{\phi=60°} mg(-\sin\phi)(12\text{ m})d\phi$$

$$= -mg(12\text{ m})\int_0^{60°}\sin\phi\,d\phi = (-80\text{ kg})(9.8\text{ m/s}^2)(12\text{ m})(-\cos\phi)\Big|_0^{60°}$$

$$= (-784\text{ N})(12\text{ m})(-\cos 60° + 1) = \boxed{-4.70\times10^3\text{ J}}$$

Method Two.

The force of gravity on Batman is $mg = (80\text{ kg})(9.8\text{ m/s}^2) = 784\text{ N}$ down. Only his vertical displacement contributes to the work gravity does. His original y-coordinate below the tree limb is –12 m. His final y-coordinate is $(-12\text{ m})\cos 60° = -6\text{ m}$. His change in elevation is $-6\text{ m} - (-12\text{ m}) = 6\text{ m}$. The work done by gravity is

$$W = F\Delta r\cos\theta = (784\text{ N})(6\text{ m})\cos 180° = \boxed{-4.70\text{ kJ}}.$$

P7.4 (a) $W = mgh = \left(3.35\times10^{-5}\right)(9.80)(100)\text{ J} = \boxed{3.28\times10^{-2}\text{ J}}$

(b) Since $R = mg$, $W_{\text{air resistance}} = \boxed{-3.28\times10^{-2}\text{ J}}$

Section 7.3 **The Scalar Product of Two Vectors**

P7.5 $A = 5.00$; $B = 9.00$; $\theta = 50.0°$

$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = (5.00)(9.00)\cos 50.0° = \boxed{28.9}$

P7.6

$$\mathbf{A}\cdot\mathbf{B} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}\right)\cdot\left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right)$$

$$\begin{aligned}\mathbf{A}\cdot\mathbf{B} &= A_xB_x\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}\right) + A_xB_y\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{j}}\right) + A_xB_z\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{k}}\right)\\ &+ A_yB_x\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{i}}\right) + A_yB_y\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{j}}\right) + A_yB_z\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{k}}\right)\\ &+ A_zB_x\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{i}}\right) + A_zB_y\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{j}}\right) + A_zB_z\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}\right)\end{aligned}$$

$$\mathbf{A}\cdot\mathbf{B} = \boxed{A_xB_x + A_yB_y + A_zB_z}$$

P7.7 (a) $W = \mathbf{F}\cdot\Delta\mathbf{r} = F_x x + F_y y = (6.00)(3.00)\text{ N}\cdot\text{m} + (-2.00)(1.00)\text{ N}\cdot\text{m} = \boxed{16.0\text{ J}}$

(b) $\theta = \cos^{-1}\left(\dfrac{\mathbf{F}\cdot\Delta\mathbf{r}}{F\Delta r}\right) = \cos^{-1}\dfrac{16}{\sqrt{\left((6.00)^2 + (-2.00)^2\right)\left((3.00)^2 + (1.00)^2\right)}} = \boxed{36.9°}$

P7.8 We must first find the angle between the two vectors. It is:

$$\theta = 360° - 118° - 90.0° - 132° = 20.0°$$

Then

$$\mathbf{F}\cdot\mathbf{v} = Fv\cos\theta = (32.8\text{ N})(0.173\text{ m/s})\cos 20.0°$$

or $\mathbf{F}\cdot\mathbf{v} = 5.33\dfrac{\text{N}\cdot\text{m}}{\text{s}} = 5.33\dfrac{\text{J}}{\text{s}} = \boxed{5.33\text{ W}}$

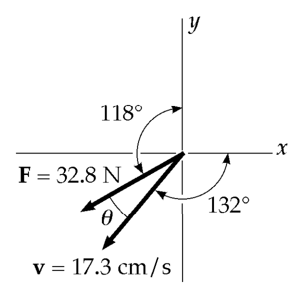

FIG. P7.8

P7.9 (a) $\mathbf{A} = 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}$

$\mathbf{B} = 4.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}$ $\qquad \theta = \cos^{-1}\dfrac{\mathbf{A}\cdot\mathbf{B}}{AB} = \cos^{-1}\dfrac{12.0 + 8.00}{\sqrt{(13.0)(32.0)}} = \boxed{11.3°}$

(b) $\mathbf{B} = 3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}} + 2.00\hat{\mathbf{k}}$

$\mathbf{A} = -2.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}$ $\qquad \cos\theta = \dfrac{\mathbf{A}\cdot\mathbf{B}}{AB} = \dfrac{-6.00 - 16.0}{\sqrt{(20.0)(29.0)}}$ $\qquad \theta = \boxed{156°}$

(c) $\mathbf{A} = \hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} + 2.00\hat{\mathbf{k}}$

$\mathbf{B} = 3.00\hat{\mathbf{j}} + 4.00\hat{\mathbf{k}}$ $\qquad \theta = \cos^{-1}\left(\dfrac{\mathbf{A}\cdot\mathbf{B}}{AB}\right) = \cos^{-1}\left(\dfrac{-6.00 + 8.00}{\sqrt{9.00}\cdot\sqrt{25.0}}\right) = \boxed{82.3°}$

P7.10 $\mathbf{A}-\mathbf{B}=\left(3.00\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}\right)-\left(-\hat{\mathbf{i}}+2.00\hat{\mathbf{j}}+5.00\hat{\mathbf{k}}\right)$

$\mathbf{A}-\mathbf{B}=4.00\hat{\mathbf{i}}-\hat{\mathbf{j}}-6.00\hat{\mathbf{k}}$

$\mathbf{C}\cdot(\mathbf{A}-\mathbf{B})=\left(2.00\hat{\mathbf{j}}-3.00\hat{\mathbf{k}}\right)\cdot\left(4.00\hat{\mathbf{i}}-\hat{\mathbf{j}}-6.00\hat{\mathbf{k}}\right)=0+(-2.00)+(+18.0)=\boxed{16.0}$

Section 7.4 **Work Done by a Varying Force**

P7.11 $W=\int_i^f F dx=$ area under curve from x_i to x_f

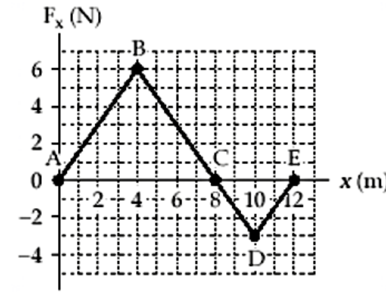

FIG. P7.11

(a) $x_i=0$ $\quad x_f=8.00$ m

$W=$ area of triangle $ABC=\left(\frac{1}{2}\right)AC\times$ altitude,

$W_{0\to 8}=\left(\frac{1}{2}\right)\times 8.00\text{ m}\times 6.00\text{ N}=\boxed{24.0\text{ J}}$

(b) $x_i=8.00$ m $\quad x_f=10.0$ m

$W=$ area of $\Delta CDE=\left(\frac{1}{2}\right)CE\times$ altitude,

$W_{8\to 10}=\left(\frac{1}{2}\right)\times(2.00\text{ m})\times(-3.00\text{ N})=\boxed{-3.00\text{ J}}$

(c) $W_{0\to 10}=W_{0\to 8}+W_{8\to 10}=24.0+(-3.00)=\boxed{21.0\text{ J}}$

P7.12 $F_x=(8x-16)$ N

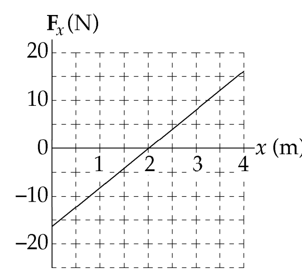

FIG. P7.12

(a) See figure to the right

(b) $W_{\text{net}}=\frac{-(2.00\text{ m})(16.0\text{ N})}{2}+\frac{(1.00\text{ m})(8.00\text{ N})}{2}=\boxed{-12.0\text{ J}}$

P7.13 $W = \int F_x dx$

and W equals the area under the Force-Displacement curve

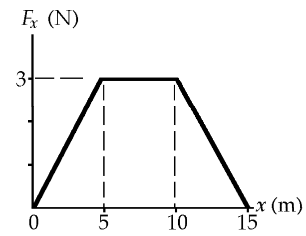

FIG. P7.13

(a) For the region $0 \le x \le 5.00$ m,

$$W = \frac{(3.00\text{ N})(5.00\text{ m})}{2} = \boxed{7.50\text{ J}}$$

(b) For the region $5.00 \le x \le 10.0$,

$$W = (3.00\text{ N})(5.00\text{ m}) = \boxed{15.0\text{ J}}$$

(c) For the region $10.0 \le x \le 15.0$,

$$W = \frac{(3.00\text{ N})(5.00\text{ m})}{2} = \boxed{7.50\text{ J}}$$

(d) For the region $0 \le x \le 15.0$

$$W = (7.50 + 7.50 + 15.0)\text{ J} = \boxed{30.0\text{ J}}$$

P7.14 $W = \int_i^f \mathbf{F} \cdot d\mathbf{r} = \int_0^{5\text{ m}} \left(4x\hat{\mathbf{i}} + 3y\hat{\mathbf{j}}\right)\text{N} \cdot dx\hat{\mathbf{i}}$

$$\int_0^{5\text{ m}} (4\text{ N/m})x\,dx + 0 = (4\text{ N/m})\left.\frac{x^2}{2}\right|_0^{5\text{ m}} = \boxed{50.0\text{ J}}$$

P7.15 $k = \frac{F}{y} = \frac{Mg}{y} = \frac{(4.00)(9.80)\text{ N}}{2.50 \times 10^{-2}\text{ m}} = 1.57 \times 10^3\text{ N/m}$

(a) For 1.50 kg mass $y = \frac{mg}{k} = \frac{(1.50)(9.80)}{1.57 \times 10^3} = \boxed{0.938\text{ cm}}$

(b) Work $= \frac{1}{2}ky^2$

Work $= \frac{1}{2}\left(1.57 \times 10^3\text{ N}\cdot\text{m}\right)\left(4.00 \times 10^{-2}\text{ m}\right)^2 = \boxed{1.25\text{ J}}$

P7.16 (a) Spring constant is given by $F = kx$

$$k = \frac{F}{x} = \frac{(230\text{ N})}{(0.400\text{ m})} = \boxed{575\text{ N/m}}$$

(b) Work $= F_{\text{avg}}x = \frac{1}{2}(230\text{ N})(0.400\text{ m}) = \boxed{46.0\text{ J}}$

***P7.17** (a)

$$F_{\text{applied}} = k_{\text{leaf}} x_\ell + k_{\text{helper}} x_h = k_\ell x_\ell + k_h (x_\ell - y_0)$$

$$5 \times 10^5 \text{ N} = 5.25 \times 10^5 \ \frac{\text{N}}{\text{m}} x_\ell + 3.60 \times 10^5 \ \frac{\text{N}}{\text{m}} (x_\ell - 0.5 \text{ m})$$

$$x_\ell = \frac{6.8 \times 10^5 \text{ N}}{8.85 \times 10^5 \text{ N/m}} = \boxed{0.768 \text{ m}}$$

(b)

$$W = \frac{1}{2} k_\ell x_\ell^2 + \frac{1}{2} k_h x_h^2 = \frac{1}{2}\left(5.25 \times 10^5 \ \frac{\text{N}}{\text{m}}\right)(0.768 \text{ m})^2 + \frac{1}{2} 3.60 \times 10^5 \ \frac{\text{N}}{\text{m}} (0.268 \text{ m})^2$$

$$= \boxed{1.68 \times 10^5 \text{ J}}$$

P7.18 (a)

$$W = \int_i^f \mathbf{F} \cdot d\mathbf{r}$$

$$W = \int_0^{0.600 \text{ m}} \left(15\,000 \text{ N} + 10\,000 x \text{ N/m} - 25\,000 x^2 \text{ N/m}^2\right) dx \cos 0°$$

$$W = 15\,000 x + \frac{10\,000 x^2}{2} - \frac{25\,000 x^3}{3} \Bigg|_0^{0.600 \text{ m}}$$

$$W = 9.00 \text{ kJ} + 1.80 \text{ kJ} - 1.80 \text{ kJ} = \boxed{9.00 \text{ kJ}}$$

(b) Similarly,

$$W = (15.0 \text{ kN})(1.00 \text{ m}) + \frac{(10.0 \text{ kN/m})(1.00 \text{ m})^2}{2} - \frac{\left(25.0 \text{ kN/m}^2\right)(1.00 \text{ m})^3}{3}$$

$W = \boxed{11.7 \text{ kJ}}$, larger by 29.6%

P7.19 $4.00 \text{ J} = \frac{1}{2} k (0.100 \text{ m})^2$

$\therefore k = 800 \text{ N/m}$ and to stretch the spring to 0.200 m requires

$$\Delta W = \frac{1}{2}(800)(0.200)^2 - 4.00 \text{ J} = \boxed{12.0 \text{ J}}$$

P7.20 (a) The radius to the object makes angle θ with the horizontal, so its weight makes angle θ with the negative side of the x-axis, when we take the x–axis in the direction of motion tangent to the cylinder.

$$\sum F_x = m a_x$$

$$F - mg \cos\theta = 0$$

$$F = \boxed{mg \cos\theta}$$

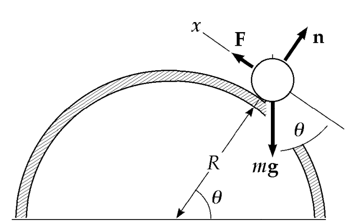

FIG. P7.20

(b)

$$W = \int_i^f \mathbf{F} \cdot d\mathbf{r}$$

We use radian measure to express the next bit of displacement as $dr = R d\theta$ in terms of the next bit of angle moved through:

$$W = \int_0^{\pi/2} mg \cos\theta R d\theta = mgR \sin\theta \Big|_0^{\pi/2}$$

$$W = mgR(1 - 0) = \boxed{mgR}$$

***P7.21** The same force makes both light springs stretch.

(a) The hanging mass moves down by

$$x = x_1 + x_2 = \frac{mg}{k_1} + \frac{mg}{k_2} = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$= 1.5 \text{ kg } 9.8 \text{ m/s}^2 \left(\frac{1 \text{ m}}{1\,200 \text{ N}} + \frac{1 \text{ m}}{1\,800 \text{ N}}\right) = \boxed{2.04 \times 10^{-2} \text{ m}}$$

(b) We define the effective spring constant as

$$k = \frac{F}{x} = \frac{mg}{mg(1/k_1 + 1/k_2)} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$$

$$= \left(\frac{1 \text{ m}}{1\,200 \text{ N}} + \frac{1 \text{ m}}{1\,800 \text{ N}}\right)^{-1} = \boxed{720 \text{ N/m}}$$

***P7.22** See the solution to problem 7.21.

(a) $x = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$

(b) $k = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$

P7.23 $[k] = \left[\frac{F}{x}\right] = \frac{\text{N}}{\text{m}} = \frac{\text{kg} \cdot \text{m/s}^2}{\text{m}} = \boxed{\frac{\text{kg}}{\text{s}^2}}$

Section 7.5 **Kinetic Energy and the Work-Kinetic Energy Theorem**

Section 7.6 **The Non-Isolated System—Conservation of Energy**

P7.24 (a) $K_A = \frac{1}{2}(0.600 \text{ kg})(2.00 \text{ m/s})^2 = \boxed{1.20 \text{ J}}$

(b) $\frac{1}{2}mv_B^2 = K_B$: $v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{(2)(7.50)}{0.600}} = \boxed{5.00 \text{ m/s}}$

(c) $\sum W = \Delta K = K_B - K_A = \frac{1}{2}m\left(v_B^2 - v_A^2\right) = 7.50 \text{ J} - 1.20 \text{ J} = \boxed{6.30 \text{ J}}$

P7.25 (a) $K = \frac{1}{2}mv^2 = \frac{1}{2}(0.300 \text{ kg})(15.0 \text{ m/s})^2 = \boxed{33.8 \text{ J}}$

(b) $K = \frac{1}{2}(0.300)(30.0)^2 = \frac{1}{2}(0.300)(15.0)^2(4) = 4(33.8) = \boxed{135 \text{ J}}$

P7.26 $\mathbf{v}_i = \left(6.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right) = \text{ m/s}$

(a) $v_i = \sqrt{v_{ix}^2 + v_{iy}^2} = \sqrt{40.0}\ \text{m/s}$

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(3.00\ \text{kg})\left(40.0\ \text{m}^2/\text{s}^2\right) = \boxed{60.0\ \text{J}}$$

(b) $\mathbf{v}_f = 8.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}$

$$v_f^2 = \mathbf{v}_f \cdot \mathbf{v}_f = 64.0 + 16.0 = 80.0\ \text{m}^2/\text{s}^2$$

$$\Delta K = K_f - K_i = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = \frac{3.00}{2}(80.0) - 60.0 = \boxed{60.0\ \text{J}}$$

P7.27 Consider the work done on the pile driver from the time it starts from rest until it comes to rest at the end of the fall. Let $d = 5.00$ m represent the distance over which the driver falls freely, and $h = 0.12$ m the distance it moves the piling.

$\sum W = \Delta K$: $\quad W_{\text{gravity}} + W_{\text{beam}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

so $\quad (mg)(h+d)\cos 0° + \left(\bar{F}\right)(d)\cos 180° = 0 - 0.$

Thus, $\quad \bar{F} = \dfrac{(mg)(h+d)}{d} = \dfrac{(2\,100\ \text{kg})\left(9.80\ \text{m/s}^2\right)(5.12\ \text{m})}{0.120\ \text{m}} = \boxed{8.78\times 10^5\ \text{N}}$. The force on the pile driver is $\boxed{\text{upward}}$.

P7.28 (a) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W = $ (area under curve from $x = 0$ to $x = 5.00$ m)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(7.50\ \text{J})}{4.00\ \text{kg}}} = \boxed{1.94\ \text{m/s}}$$

(b) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W = $ (area under curve from $x = 0$ to $x = 10.0$ m)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(22.5\ \text{J})}{4.00\ \text{kg}}} = \boxed{3.35\ \text{m/s}}$$

(c) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W = $ (area under curve from $x = 0$ to $x = 15.0$ m)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(30.0\ \text{J})}{4.00\ \text{kg}}} = \boxed{3.87\ \text{m/s}}$$

P7.29 (a) $K_i + \sum W = K_f = \frac{1}{2}mv_f^2$

$$0 + \sum W = \frac{1}{2}\left(15.0\times 10^{-3}\ \text{kg}\right)(780\ \text{m/s})^2 = \boxed{4.56\ \text{kJ}}$$

(b) $$F = \frac{W}{\Delta r \cos\theta} = \frac{4.56\times 10^3\ \text{J}}{(0.720\ \text{m})\cos 0°} = \boxed{6.34\ \text{kN}}$$

(c) $$a = \frac{v_f^2 - v_i^2}{2x_f} = \frac{(780\ \text{m/s})^2 - 0}{2(0.720\ \text{m})} = \boxed{422\ \text{km/s}^2}$$

(d) $$\sum F = ma = \left(15\times 10^{-3}\ \text{kg}\right)\left(422\times 10^3\ \text{m/s}^2\right) = \boxed{6.34\ \text{kN}}$$

P7.30 (a) $v_f = 0.096\left(3\times10^8 \text{ m/s}\right) = 2.88\times10^7 \text{ m/s}$

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}\left(9.11\times10^{-31}\text{ kg}\right)\left(2.88\times10^7\text{ m/s}\right)^2 = \boxed{3.78\times10^{-16}\text{ J}}$$

(b) $K_i + W = K_f$: $\quad 0 + F\Delta r\cos\theta = K_f$

$$F(0.028\text{ m})\cos 0° = 3.78\times10^{-16}\text{ J}$$

$$F = \boxed{1.35\times10^{-14}\text{ N}}$$

(c) $\sum F = ma$; $\quad a = \frac{\sum F}{m} = \frac{1.35\times10^{-14}\text{ N}}{9.11\times10^{-31}\text{ kg}} = \boxed{1.48\times10^{+16}\text{ m/s}^2}$

(d) $v_{xf} = v_{xi} + a_x t \quad 2.88\times10^7\text{ m/s} = 0 + \left(1.48\times10^{16}\text{ m/s}^2\right)t$

$$t = \boxed{1.94\times10^{-9}\text{ s}}$$

Check: $\quad x_f = x_i + \frac{1}{2}\left(v_{xi} + v_{xf}\right)t$

$$0.028\text{ m} = 0 + \frac{1}{2}\left(0 + 2.88\times10^7\text{ m/s}\right)t$$

$$t = 1.94\times10^{-9}\text{ s}$$

Section 7.7 **Situations Involving Kinetic Friction**

P7.31 $\sum F_y = ma_y$: $\quad n - 392\text{ N} = 0$

$$n = 392\text{ N}$$

$$f_k = \mu_k n = (0.300)(392\text{ N}) = 118\text{ N}$$

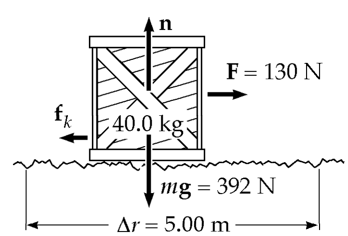

FIG. P7.31

(a) $W_F = F\Delta r\cos\theta = (130)(5.00)\cos 0° = \boxed{650\text{ J}}$

(b) $\Delta E_{\text{int}} = f_k\Delta x = (118)(5.00) = \boxed{588\text{ J}}$

(c) $W_n = n\Delta r\cos\theta = (392)(5.00)\cos 90° = \boxed{0}$

(d) $W_g = mg\Delta r\cos\theta = (392)(5.00)\cos(-90°) = \boxed{0}$

(e) $\Delta K = K_f - K_i = \sum W_{\text{other}} - \Delta E_{\text{int}}$

$$\frac{1}{2}mv_f^2 - 0 = 650\text{ J} - 588\text{ J} + 0 + 0 = \boxed{62.0\text{ J}}$$

(f) $v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(62.0\text{ J})}{40.0\text{ kg}}} = \boxed{1.76\text{ m/s}}$

P7.32 (a) $W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = \frac{1}{2}(500)\left(5.00\times10^{-2}\right)^2 - 0 = 0.625\ \text{J}$

$W_s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 - 0$

so $v_f = \sqrt{\frac{2\left(\sum W\right)}{m}} = \sqrt{\frac{2(0.625)}{2.00}}\ \text{m/s} = \boxed{0.791\ \text{m/s}}$

(b) $\frac{1}{2}mv_i^2 - f_k\Delta x + W_s = \frac{1}{2}mv_f^2$

$0 - (0.350)(2.00)(9.80)(0.050\,0)\ \text{J} + 0.625\ \text{J} = \frac{1}{2}mv_f^2$

$0.282\ \text{J} = \frac{1}{2}(2.00\ \text{kg})v_f^2$

$v_f = \sqrt{\frac{2(0.282)}{2.00}}\ \text{m/s} = \boxed{0.531\ \text{m/s}}$

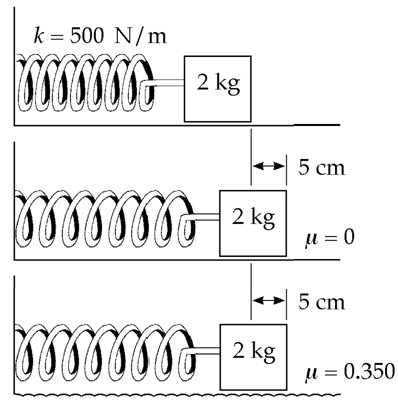

FIG. P7.32

P7.33 (a) $W_g = mg\ell\cos(90.0° + \theta)$

$W_g = (10.0\ \text{kg})\left(9.80\ \text{m/s}^2\right)(5.00\ \text{m})\cos 110° = \boxed{-168\ \text{J}}$

(b) $f_k = \mu_k n = \mu_k mg\cos\theta$

$\Delta E_{\text{int}} = \ell f_k = \ell\mu_k mg\cos\theta$

$\Delta E_{\text{int}} = (5.00\ \text{m})(0.400)(10.0)(9.80)\cos 20.0° = \boxed{184\ \text{J}}$

(c) $W_F = F\ell = (100)(5.00) = \boxed{500\ \text{J}}$

(d) $\Delta K = \sum W_{\text{other}} - \Delta E_{\text{int}} = W_F + W_g - \Delta E_{\text{int}} = \boxed{148\ \text{J}}$

(e) $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$v_f = \sqrt{\frac{2(\Delta K)}{m} + v_i^2} = \sqrt{\frac{2(148)}{10.0} + (1.50)^2} = \boxed{5.65\ \text{m/s}}$

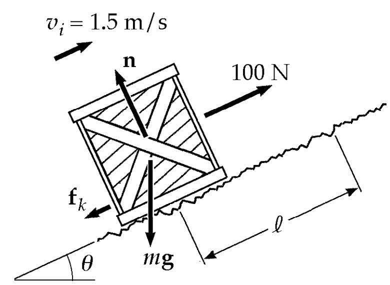

FIG. P7.33

P7.34 $\sum F_y = ma_y$: $n + (70.0\ \text{N})\sin 20.0° - 147\ \text{N} = 0$

$n = 123\ \text{N}$

$f_k = \mu_k n = 0.300(123\ \text{N}) = 36.9\ \text{N}$

(a) $W = F\Delta r\cos\theta = (70.0\ \text{N})(5.00\ \text{m})\cos 20.0° = \boxed{329\ \text{J}}$

(b) $W = F\Delta r\cos\theta = (123\ \text{N})(5.00\ \text{m})\cos 90.0° = \boxed{0\ \text{J}}$

(c) $W = F\Delta r\cos\theta = (147\ \text{N})(5.00\ \text{m})\cos 90.0° = \boxed{0}$

(d) $\Delta E_{\text{int}} = F\Delta x = (36.9\ \text{N})(5.00\ \text{m}) = \boxed{185\ \text{J}}$

(e) $\Delta K = K_f - K_i = \sum W - \Delta E_{\text{int}} = 329\ \text{J} - 185\ \text{J} = \boxed{+144\ \text{J}}$

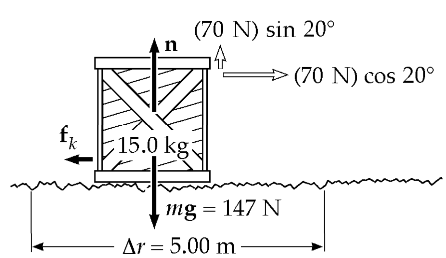

FIG. P7.34

P7.35 $v_i = 2.00\text{ m/s}$ $\mu_k = 0.100$

$K_i - f_k\Delta x + W_{\text{other}} = K_f$: $\frac{1}{2}mv_i^2 - f_k\Delta x = 0$

$\frac{1}{2}mv_i^2 = \mu_k mg\Delta x$ $\Delta x = \frac{v_i^2}{2\mu_k g} = \frac{(2.00\text{ m/s})^2}{2(0.100)(9.80)} = \boxed{2.04\text{ m}}$

Section 7.8 **Power**

***P7.36** $$\mathcal{P}_{\text{av}} = \frac{W}{\Delta t} = \frac{K_f}{\Delta t} = \frac{mv^2}{2\Delta t} = \frac{0.875\text{ kg}(0.620\text{ m/s})^2}{2\left(21\times 10^{-3}\text{ s}\right)} = \boxed{8.01\text{ W}}$$

P7.37 $\text{Power} = \frac{W}{t}$ $\mathcal{P} = \frac{mgh}{t} = \frac{(700\text{ N})(10.0\text{ m})}{8.00\text{ s}} = \boxed{875\text{ W}}$

P7.38 A 1 300-kg car speeds up from rest to 55.0 mi/h = 24.6 m/s in 15.0 s. The output work of the engine is equal to its final kinetic energy,

$$\frac{1}{2}(1\,300\text{ kg})(24.6\text{ m/s})^2 = 390\text{ kJ}$$

with power $\mathcal{P} = \frac{390\,000\text{ J}}{15.0\text{ s}}\boxed{\sim 10^4\text{ W}}$ around 30 horsepower.

P7.39 (a) $\sum W = \Delta K$, but $\Delta K = 0$ because he moves at constant speed. The skier rises a vertical distance of $(60.0\text{ m})\sin 30.0° = 30.0\text{ m}$. Thus,

$$W_{\text{in}} = -W_g = (70.0\text{ kg})\left(9.8\text{ m/s}^2\right)(30.0\text{ m}) = \boxed{2.06\times 10^4\text{ J}} = \boxed{20.6\text{ kJ}}.$$

(b) The time to travel 60.0 m at a constant speed of 2.00 m/s is 30.0 s. Thus,

$$\mathcal{P}_{\text{input}} = \frac{W}{\Delta t} = \frac{2.06\times 10^4\text{ J}}{30.0\text{ s}} = \boxed{686\text{ W}} = 0.919\text{ hp}.$$

P7.40 (a) The distance moved upward in the first 3.00 s is

$$\Delta y = \bar{v}t = \left[\frac{0 + 1.75\text{ m/s}}{2}\right](3.00\text{ s}) = 2.63\text{ m}.$$

The motor and the earth's gravity do work on the elevator car:

$$\frac{1}{2}mv_i^2 + W_{\text{motor}} + mg\Delta y\cos 180° = \frac{1}{2}mv_f^2$$

$$W_{\text{motor}} = \frac{1}{2}(650\text{ kg})(1.75\text{ m/s})^2 - 0 + (650\text{ kg})g(2.63\text{ m}) = 1.77\times 10^4\text{ J}$$

Also, $W = \bar{\mathcal{P}}t$ so $\bar{\mathcal{P}} = \frac{W}{t} = \frac{1.77\times 10^4\text{ J}}{3.00\text{ s}} = \boxed{5.91\times 10^3\text{ W}} = 7.92\text{ hp}.$

(b) When moving upward at constant speed $(v = 1.75\text{ m/s})$ the applied force equals the weight $= (650\text{ kg})\left(9.80\text{ m/s}^2\right) = 6.37\times 10^3\text{ N}$. Therefore,

$$\mathcal{P} = Fv = \left(6.37\times 10^3\text{ N}\right)(1.75\text{ m/s}) = \boxed{1.11\times 10^4\text{ W}} = 14.9\text{ hp}.$$

P7.41 *energy* = *power* × *time*

For the 28.0 W bulb:

$$\text{Energy used} = (28.0\text{ W})\left(1.00\times10^{4}\text{ h}\right) = 280\text{ kilowatt}\cdot\text{hrs}$$
$$\text{total cost} = \$17.00 + (280\text{ kWh})(\$0.080/\text{kWh}) = \$39.40$$

For the 100 W bulb:

$$\text{Energy used} = (100\text{ W})\left(1.00\times10^{4}\text{ h}\right) = 1.00\times10^{3}\text{ kilowatt}\cdot\text{hrs}$$
$$\#\text{ bulb used} = \frac{1.00\times10^{4}\text{ h}}{750\text{ h/bulb}} = 13.3$$
$$\text{total cost} = 13.3(\$0.420) + \left(1.00\times10^{3}\text{ kWh}\right)(\$0.080/\text{kWh}) = \$85.60$$

Savings with energy-efficient bulb $= \$85.60 - \$39.40 = \boxed{\$46.20}$

***P7.42** (a) Burning 1 lb of fat releases energy $1\text{ lb}\left(\frac{454\text{ g}}{1\text{ lb}}\right)\left(\frac{9\text{ kcal}}{1\text{ g}}\right)\left(\frac{4\,186\text{ J}}{1\text{ kcal}}\right) = 1.71\times10^{7}\text{ J}.$

The mechanical energy output is $\left(1.71\times10^{7}\text{ J}\right)(0.20) = nF\Delta r\cos\theta.$

Then
$$3.42\times10^{6}\text{ J} = nmg\Delta y\cos 0°$$
$$3.42\times10^{6}\text{ J} = n(50\text{ kg})\left(9.8\text{ m/s}^2\right)(80\text{ steps})(0.150\text{ m})$$
$$3.42\times10^{6}\text{ J} = n\left(5.88\times10^{3}\text{ J}\right)$$

where the number of times she must climb the steps is $n = \frac{3.42\times10^{6}\text{ J}}{5.88\times10^{3}\text{ J}} = \boxed{582}.$

This method is impractical compared to limiting food intake.

(b) Her mechanical power output is

$$\mathcal{P} = \frac{W}{t} = \frac{5.88\times10^{3}\text{ J}}{65\text{ s}} = \boxed{90.5\text{ W}} = 90.5\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right) = \boxed{0.121\text{ hp}}.$$

***P7.43** (a) The fuel economy for walking is $\frac{1\text{ h}}{220\text{ kcal}}\left(\frac{3\text{ mi}}{\text{h}}\right)\left(\frac{1\text{ kcal}}{4\,186\text{ J}}\right)\left(\frac{1.30\times10^{8}\text{ J}}{1\text{ gal}}\right) = \boxed{423\text{ mi/gal}}.$

(b) For bicycling $\frac{1\text{ h}}{400\text{ kcal}}\left(\frac{10\text{ mi}}{\text{h}}\right)\left(\frac{1\text{ kcal}}{4\,186\text{ J}}\right)\left(\frac{1.30\times10^{8}\text{ J}}{1\text{ gal}}\right) = \boxed{776\text{ mi/gal}}.$

Section 7.9 **Energy and the Automobile**

P7.44 At a speed of 26.8 m/s (60.0 mph), the car described in Table 7.2 delivers a power of $\mathscr{P}_1 = 18.3 \text{ kW}$ to the wheels. If an additional load of 350 kg is added to the car, a larger output power of

$$\mathscr{P}_2 = \mathscr{P}_1 + (\text{power input to move 350 kg at speed } v)$$

will be required. The additional power output needed to move 350 kg at speed v is:

$$\Delta\mathscr{P}_{\text{out}} = (\Delta f)v = (\mu_r mg)v.$$

Assuming a coefficient of rolling friction of $\mu_r = 0.016\,0$, the power output now needed from the engine is

$$\mathscr{P}_2 = \mathscr{P}_1 + (0.016\,0)(350 \text{ kg})(9.80 \text{ m/s}^2)(26.8 \text{ m/s}) = 18.3 \text{ kW} + 1.47 \text{ kW}.$$

With the assumption of constant efficiency of the engine, the input power must increase by the same factor as the output power. Thus, the fuel economy must decrease by this factor:

$$(\text{fuel economy})_2 = \left(\frac{\mathscr{P}_1}{\mathscr{P}_2}\right)(\text{fuel economy})_1 = \left(\frac{18.3}{18.3+1.47}\right)(6.40 \text{ km/L})$$

or $(\text{fuel economy})_2 = \boxed{5.92 \text{ km/L}}$.

P7.45 (a)

$$\text{fuel needed} = \frac{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}{\text{useful energy per gallon}} = \frac{\frac{1}{2}mv_f^2 - 0}{\text{eff.}\times(\text{energy content of fuel})}$$

$$= \frac{\frac{1}{2}(900 \text{ kg})(24.6 \text{ m/s})^2}{(0.150)(1.34\times 10^8 \text{ J/gal})} = \boxed{1.35\times 10^{-2} \text{ gal}}$$

(b) $\boxed{73.8}$

(c)

$$\text{power} = \left(\frac{1 \text{ gal}}{38.0 \text{ mi}}\right)\left(\frac{55.0 \text{ mi}}{1.00 \text{ h}}\right)\left(\frac{1.00 \text{ h}}{3\,600 \text{ s}}\right)\left(\frac{1.34\times 10^8 \text{ J}}{1 \text{ gal}}\right)(0.150) = \boxed{8.08 \text{ kW}}$$

Additional Problems

P7.46 At start, $\mathbf{v} = (40.0 \text{ m/s})\cos 30.0°\,\hat{\mathbf{i}} + (40.0 \text{ m/s})\sin 30.0°\,\hat{\mathbf{j}}$

At apex, $\mathbf{v} = (40.0 \text{ m/s})\cos 30.0°\,\hat{\mathbf{i}} + 0\hat{\mathbf{j}} = (34.6 \text{ m/s})\hat{\mathbf{i}}$

And $K = \frac{1}{2}mv^2 = \frac{1}{2}(0.150 \text{ kg})(34.6 \text{ m/s})^2 = \boxed{90.0 \text{ J}}$

P7.47 Concentration of Energy output $= (0.600\ \text{J/kg}\cdot\text{step})(60.0\ \text{kg})\left(\dfrac{1\ \text{step}}{1.50\ \text{m}}\right) = 24.0\ \text{J/m}$

$$F = (24.0\ \text{J/m})(1\ \text{N}\cdot\text{m/J}) = 24.0\ \text{N}$$
$$\mathscr{P} = Fv$$
$$70.0\ \text{W} = (24.0\ \text{N})v$$
$$v = \boxed{2.92\ \text{m/s}}$$

P7.48 (a) $\mathbf{A}\cdot\hat{\mathbf{i}} = (A)(1)\cos\alpha$. But also, $\mathbf{A}\cdot\hat{\mathbf{i}} = A_x$.

Thus, $(A)(1)\cos\alpha = A_x$ or $\boxed{\cos\alpha = \dfrac{A_x}{A}}$.

Similarly, $\boxed{\cos\beta = \dfrac{A_y}{A}}$

and $\boxed{\cos\gamma = \dfrac{A_z}{A}}$

where $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$.

(b) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \left(\dfrac{A_x}{A}\right)^2 + \left(\dfrac{A_y}{A}\right)^2 + \left(\dfrac{A_z}{A}\right)^2 = \dfrac{A^2}{A^2} = 1$

P7.49 (a) $x = t + 2.00t^3$

Therefore,

$$v = \frac{dx}{dt} = 1 + 6.00t^2$$
$$K = \frac{1}{2}mv^2 = \frac{1}{2}(4.00)\left(1 + 6.00t^2\right)^2 = \boxed{\left(2.00 + 24.0t^2 + 72.0t^4\right)\ \text{J}}$$

(b) $a = \dfrac{dv}{dt} = \boxed{(12.0t)\ \text{m/s}^2}$

$F = ma = 4.00(12.0t) = \boxed{(48.0t)\ \text{N}}$

(c) $\mathscr{P} = Fv = (48.0t)\left(1 + 6.00t^2\right) = \boxed{\left(48.0t + 288t^3\right)\ \text{W}}$

(d) $W = \displaystyle\int_0^{2.00} \mathscr{P}dt = \int_0^{2.00}\left(48.0t + 288t^3\right)dt = \boxed{1\,250\ \text{J}}$

***P7.50** (a) We write

$$F = ax^b$$

$$1\,000 \text{ N} = a(0.129 \text{ m})^b$$

$$5\,000 \text{ N} = a(0.315 \text{ m})^b$$

$$5 = \left(\frac{0.315}{0.129}\right)^b = 2.44^b$$

$$\ln 5 = b \ln 2.44$$

$$b = \frac{\ln 5}{\ln 2.44} = \boxed{1.80 = b}$$

$$a = \frac{1\,000 \text{ N}}{(0.129 \text{ m})^{1.80}} = \boxed{4.01 \times 10^4 \text{ N/m}^{1.8} = a}$$

(b)

$$W = \int_0^{0.25 \text{ m}} F dx = \int_0^{0.25 \text{ m}} 4.01 \times 10^4 \frac{\text{N}}{\text{m}^{1.8}} x^{1.8} dx$$

$$= 4.01 \times 10^4 \frac{\text{N}}{\text{m}^{1.8}} \left.\frac{x^{2.8}}{2.8}\right|_0^{0.25 \text{ m}} = 4.01 \times 10^4 \frac{\text{N}}{\text{m}^{1.8}} \frac{(0.25 \text{ m})^{2.8}}{2.8}$$

$$= \boxed{294 \text{ J}}$$

***P7.51** The work done by the applied force is

$$W = \int_i^f F_{\text{applied}} dx = \int_0^{x_{\max}} -\left[-\left(k_1 x + k_2 x^2\right)\right] dx$$

$$= \int_0^{x_{\max}} k_1 x \, dx + \int_0^{x_{\max}} k_2 x^2 dx = k_1 \left.\frac{x^2}{2}\right|_0^{x_{\max}} + k_2 \left.\frac{x^3}{3}\right|_0^{x_{\max}}$$

$$= \boxed{k_1 \frac{x_{\max}^2}{2} + k_2 \frac{x_{\max}^3}{3}}$$

P7.52 (a) The work done by the traveler is $mgh_s N$ where N is the number of steps he climbs during the ride.

$$N = (\text{time on escalator})(n)$$

where

$$(\text{time on escalator}) = \frac{h}{\text{vertical velocity of person}}$$

and

$$\text{vertical velocity of person} = v + nh_s$$

Then,

$$N = \frac{nh}{v + nh_s}$$

and the work done by the person becomes $W_{\text{person}} = \boxed{\dfrac{mgnhh_s}{v + nh_s}}$

continued on next page

(b) The work done by the escalator is

$$W_e = (\text{power})(\text{time}) = \left[(\text{force exerted})(\text{speed})(\text{time})\right] = mgvt$$

where $t = \dfrac{h}{v + nh_s}$ as above.

Thus, $W_e = \boxed{\dfrac{mgvh}{v + nh_s}}$.

As a check, the total work done on the person's body must add up to *mgh*, the work an elevator would do in lifting him.

It does add up as follows: $\sum W = W_{\text{person}} + W_e = \dfrac{mgnhh_s}{v + nh_s} + \dfrac{mgvh}{v + nh_s} = \dfrac{mgh(nh_s + v)}{v + nh_s} = mgh$

P7.53 (a) $\Delta K = \dfrac{1}{2}mv^2 - 0 = \sum W$, so

$v^2 = \dfrac{2W}{m}$ and $v = \boxed{\sqrt{\dfrac{2W}{m}}}$

(b) $W = \mathbf{F} \cdot \mathbf{d} = F_x d \Rightarrow F_x = \boxed{\dfrac{W}{d}}$

***P7.54** During its whole motion from $y = 10.0\text{ m}$ to $y = -3.20\text{ mm}$, the force of gravity and the force of the plate do work on the ball. It starts and ends at rest

$$K_i + \sum W = K_f$$

$$0 + F_g \Delta y \cos 0° + F_p \Delta x \cos 180° = 0$$

$$mg(10.003\ 2\text{ m}) - F_p(0.003\ 20\text{ m}) = 0$$

$$F_p = \frac{5\text{ kg}(9.8\text{ m/s}^2)(10\text{ m})}{3.2 \times 10^{-3}\text{ m}} = \boxed{1.53 \times 10^5\text{ N upward}}$$

P7.55 (a) $\mathscr{P} = Fv = F(v_i + at) = F\left(0 + \dfrac{F}{m}t\right) = \boxed{\left(\dfrac{F^2}{m}\right)t}$

(b) $\mathscr{P} = \left[\dfrac{(20.0\text{ N})^2}{5.00\text{ kg}}\right](3.00\text{ s}) = \boxed{240\text{ W}}$

***P7.56** (a) $$W_1=\int_i^f F_1dx=\int_{x_{i1}}^{x_{i1}+x_a}k_1x\,dx=\frac{1}{2}k_1\left[\left(x_{i1}+x_a\right)^2-x_{i1}^2\right]=\frac{1}{2}k_1\left(x_a^2+2x_ax_{i1}\right)$$

(b) $$W_2=\int_{-x_{i2}}^{-x_{i2}+x_a}k_2x\,dx=\frac{1}{2}k_2\left[\left(-x_{i2}+x_a\right)^2-x_{i2}^2\right]=\frac{1}{2}k_2\left(x_a^2-2x_ax_{i2}\right)$$

(c) Before the horizontal force is applied, the springs exert equal forces: $k_1x_{i1}=k_2x_{i2}$

$$x_{i2}=\frac{k_1x_{i1}}{k_2}$$

(d) $$W_1+W_2=\frac{1}{2}k_1x_a^2+k_1x_ax_{i1}+\frac{1}{2}k_2x_a^2-k_2x_ax_{i2}$$
$$=\frac{1}{2}k_1x_a^2+\frac{1}{2}k_2x_a^2+k_1x_ax_{i1}-k_2x_a\frac{k_1x_{i1}}{k_2}$$
$$=\frac{1}{2}\left(k_1+k_2\right)x_a^2$$

***P7.57** (a) $$v=\int_0^t a\,dt=\int_0^t\left(1.16t-0.21t^2+0.24t^3\right)dt$$
$$=1.16\frac{t^2}{2}-0.21\frac{t^3}{3}+0.24\frac{t^4}{4}\bigg|_0^t=0.58t^2-0.07t^3+0.06t^4$$

At $t=0$, $v_i=0$. At $t=2.5\text{ s}$,

$$v_f=\left(0.58\text{ m/s}^3\right)(2.5\text{ s})^2-\left(0.07\text{ m/s}^4\right)(2.5\text{ s})^3+\left(0.06\text{ m/s}^5\right)(2.5\text{ s})^4=4.88\text{ m/s}$$
$$K_i+W=K_f$$
$$0+W=\frac{1}{2}mv_f^2=\frac{1}{2}1\,160\text{ kg}\left(4.88\text{ m/s}\right)^2=\boxed{1.38\times10^4\text{ J}}$$

(b) At $t=2.5\text{ s}$,

$$a=\left(1.16\text{ m/s}^3\right)2.5\text{ s}-\left(0.210\text{ m/s}^4\right)(2.5\text{ s})^2+\left(0.240\text{ m/s}^5\right)(2.5\text{ s})^3=5.34\text{ m/s}^2.$$

Through the axles the wheels exert on the chassis force

$$\sum F=ma=1\,160\text{ kg }5.34\text{ m/s}^2=6.19\times10^3\text{ N}$$

and inject power

$$\mathcal{P}=Fv=6.19\times10^3\text{ N}\left(4.88\text{ m/s}\right)=\boxed{3.02\times10^4\text{ W}}.$$

P7.58 (a) The new length of each spring is $\sqrt{x^2+L^2}$, so its extension is $\sqrt{x^2+L^2}-L$ and the force it exerts is $k\left(\sqrt{x^2+L^2}-L\right)$ toward its fixed end. The y components of the two spring forces add to zero. Their x components add to

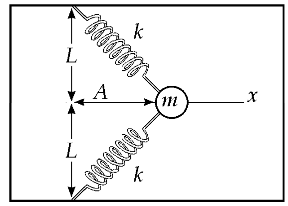

FIG. P7.58

$$\mathbf{F}=-2\hat{\mathbf{i}}k\left(\sqrt{x^2+L^2}-L\right)\frac{x}{\sqrt{x^2+L^2}}=\boxed{-2kx\hat{\mathbf{i}}\left(1-\frac{L}{\sqrt{x^2+L^2}}\right)}.$$

(b)

$$W=\int_i^f F_x dx \qquad W=\int_A^0 -2kx\left(1-\frac{L}{\sqrt{x^2+L^2}}\right)dx$$

$$W=-2k\int_A^0 x\,dx+kL\int_A^0\left(x^2+L^2\right)^{-1/2}2x\,dx \qquad W=-2k\left.\frac{x^2}{2}\right|_A^0+kL\left.\frac{\left(x^2+L^2\right)^{1/2}}{(1/2)}\right|_A^0$$

$$W=-0+kA^2+2kL^2-2kL\sqrt{A^2+L^2} \qquad W=\boxed{2kL^2+kA^2-2kL\sqrt{A^2+L^2}}$$

***P7.59** For the rocket falling at terminal speed we have

$$\sum F=ma$$
$$+R-Mg=0$$
$$Mg=\frac{1}{2}D\rho Av_T^2$$

(a) For the rocket with engine exerting thrust T and flying up at the same speed,

$$\sum F=ma$$
$$+T-Mg-R=0$$
$$T=2Mg$$

The engine power is $\mathscr{P}=Fv=Tv_T=\boxed{2Mgv_T}$.

(b) For the rocket with engine exerting thrust T_b and flying down steadily at $3v_T$,

$R_b=\frac{1}{2}D\rho A(3v_T)^2=9Mg$

$$\sum F=ma$$
$$-T_b-Mg+9Mg=0$$
$$T_b=8Mg$$

The engine power is $\mathscr{P}=Tv=8Mg3v_T=\boxed{24Mgv_T}$.

P7.60 (a) $\mathbf{F}_1 = (25.0\text{ N})\left(\cos 35.0°\,\hat{\mathbf{i}} + \sin 35.0°\,\hat{\mathbf{j}}\right) = \boxed{\left(20.5\hat{\mathbf{i}} + 14.3\hat{\mathbf{j}}\right)\text{ N}}$

$\mathbf{F}_2 = (42.0\text{ N})\left(\cos 150°\,\hat{\mathbf{i}} + \sin 150°\,\hat{\mathbf{j}}\right) = \boxed{\left(-36.4\hat{\mathbf{i}} + 21.0\hat{\mathbf{j}}\right)\text{ N}}$

(b) $\sum \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \boxed{\left(-15.9\hat{\mathbf{i}} + 35.3\hat{\mathbf{j}}\right)\text{ N}}$

(c) $\mathbf{a} = \dfrac{\sum \mathbf{F}}{m} = \boxed{\left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)\text{ m/s}^2}$

(d) $\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = \left(4.00\hat{\mathbf{i}} + 2.50\hat{\mathbf{j}}\right)\text{ m/s} + \left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)\left(\text{m/s}^2\right)(3.00\text{ s})$

$\mathbf{v}_f = \boxed{\left(-5.54\hat{\mathbf{i}} + 23.7\hat{\mathbf{j}}\right)\text{ m/s}}$

(e) $\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \dfrac{1}{2}\mathbf{a}t^2$

$\mathbf{r}_f = 0 + \left(4.00\hat{\mathbf{i}} + 2.50\hat{\mathbf{j}}\right)(\text{m/s})(3.00\text{ s}) + \dfrac{1}{2}\left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)\left(\text{m/s}^2\right)(3.00\text{ s})^2$

$\Delta\mathbf{r} = \mathbf{r}_f = \boxed{\left(-2.30\hat{\mathbf{i}} + 39.3\hat{\mathbf{j}}\right)\text{ m}}$

(f) $K_f = \dfrac{1}{2}mv_f^2 = \dfrac{1}{2}(5.00\text{ kg})\left[(5.54)^2 + (23.7)^2\right]\left(\text{m/s}^2\right) = \boxed{1.48\text{ kJ}}$

(g) $K_f = \dfrac{1}{2}mv_i^2 + \sum \mathbf{F}\cdot\Delta\mathbf{r}$

$K_f = \dfrac{1}{2}(5.00\text{ kg})\left[(4.00)^2 + (2.50)^2\right](\text{m/s})^2 + \left[(-15.9\text{ N})(-2.30\text{ m}) + (35.3\text{ N})(39.3\text{ m})\right]$

$K_f = 55.6\text{ J} + 1\,426\text{ J} = \boxed{1.48\text{ kJ}}$

P7.61 (a) $\sum W = \Delta K: \qquad W_s + W_g = 0$

$\dfrac{1}{2}kx_i^2 - 0 + mg\Delta x\cos(90°+60°) = 0$

$\dfrac{1}{2}\left(1.40\times10^3\text{ N/m}\right)\times(0.100)^2 - (0.200)(9.80)(\sin 60.0°)\Delta x = 0$

$\Delta x = \boxed{4.12\text{ m}}$

(b) $\sum W = \Delta K + \Delta E_{\text{int}}: \qquad W_s + W_g - \Delta E_{\text{int}} = 0$

$\dfrac{1}{2}kx_i^2 + mg\Delta x\cos 150° - \mu_k mg\cos 60°\,\Delta x = 0$

$\dfrac{1}{2}\left(1.40\times10^3\text{ N/m}\right)\times(0.100)^2 - (0.200)(9.80)(\sin 60.0°)\Delta x - (0.200)(9.80)(0.400)(\cos 60.0°)\Delta x = 0$

$\Delta x = \boxed{3.35\text{ m}}$

P7.62 (a)

F(N)	L(mm)	F(N)	L(mm)
2.00	15.0	14.0	112
4.00	32.0	16.0	126
6.00	49.0	18.0	149
8.00	64.0	20.0	175
10.0	79.0	22.0	190
12.0	98.0		

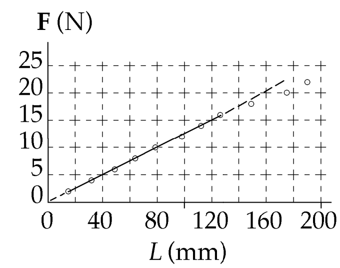

FIG. P7.62

(b) A straight line fits the first eight points, together with the origin. By least-square fitting, its slope is

$$0.125\ \text{N/mm} \pm 2\% = \boxed{125\ \text{N/m}} \pm 2\%$$

In $F = kx$, the spring constant is $k = \dfrac{F}{x}$, the same as the slope of the F-versus-x graph.

(c) $F = kx = (125\ \text{N/m})(0.105\ \text{m}) = \boxed{13.1\ \text{N}}$

P7.63

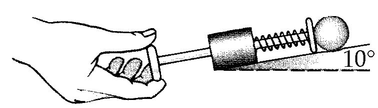

FIG. P7.63

$$K_i + W_s + W_g = K_f$$

$$\frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 + mg\Delta x\cos\theta = \frac{1}{2}mv_f^2$$

$$0 + \frac{1}{2}kx_i^2 - 0 + mgx_i\cos 100° = \frac{1}{2}mv_f^2$$

$$\frac{1}{2}(1.20\ \text{N/cm})(5.00\ \text{cm})(0.050\,0\ \text{m}) - (0.100\ \text{kg})\left(9.80\ \text{m/s}^2\right)(0.050\,0\ \text{m})\sin 10.0° = \frac{1}{2}(0.100\ \text{kg})v^2$$

$$0.150\ \text{J} - 8.51\times10^{-3}\ \text{J} = (0.050\,0\ \text{kg})v^2$$

$$v = \sqrt{\frac{0.141}{0.050\,0}} = \boxed{1.68\ \text{m/s}}$$

P7.64 (a) $\Delta E_{\text{int}} = -\Delta K = -\dfrac{1}{2}m\left(v_f^2 - v_i^2\right)$: $\Delta E_{\text{int}} = -\dfrac{1}{2}(0.400\ \text{kg})\left((6.00)^2 - (8.00)^2\right)(\text{m/s})^2 = \boxed{5.60\ \text{J}}$

(b) $\Delta E_{\text{int}} = f\Delta r = \mu_k mg(2\pi r)$: $5.60\ \text{J} = \mu_k(0.400\ \text{kg})\left(9.80\ \text{m/s}^2\right)2\pi(1.50\ \text{m})$

Thus, $\mu_k = \boxed{0.152}$.

(c) After N revolutions, the object comes to rest and $K_f = 0$.

Thus, $\Delta E_{\text{int}} = -\Delta K = -0 + K_i = \dfrac{1}{2}mv_i^2$

or $\mu_k mg\left[N(2\pi r)\right] = \dfrac{1}{2}mv_i^2$.

This gives $N = \dfrac{\frac{1}{2}mv_i^2}{\mu_k mg(2\pi r)} = \dfrac{\frac{1}{2}(8.00\ \text{m/s})^2}{(0.152)\left(9.80\ \text{m/s}^2\right)2\pi(1.50\ \text{m})} = \boxed{2.28\ \text{rev}}$.

P7.65 If positive F represents an outward force, (same as direction as r), then

$$W = \int_i^f \mathbf{F}\cdot d\mathbf{r} = \int_{r_i}^{r_f} \left(2F_0\sigma^{13}r^{-13} - F_0\sigma^7 r^{-7}\right)dr$$

$$W = \left.\frac{2F_0\sigma^{13}r^{-12}}{-12} - \frac{F_0\sigma^7 r^{-6}}{-6}\right|_{r_i}^{r_f}$$

$$W = \frac{-F_0\sigma^{13}\left(r_f^{-12} - r_i^{-12}\right)}{6} + \frac{F_0\sigma^7\left(r_f^{-6} - r_i^{-6}\right)}{6} = \frac{F_0\sigma^7}{6}\left[r_f^{-6} - r_i^{-6}\right] - \frac{F_0\sigma^{13}}{6}\left[r_f^{-12} - r_i^{-12}\right]$$

$$W = 1.03\times10^{-77}\left[r_f^{-6} - r_i^{-6}\right] - 1.89\times10^{-134}\left[r_f^{-12} - r_i^{-12}\right]$$

$$W = 1.03\times10^{-77}\left[1.88\times10^{-6} - 2.44\times10^{-6}\right]10^{60} - 1.89\times10^{-134}\left[3.54\times10^{-12} - 5.96\times10^{-8}\right]10^{120}$$

$$W = -2.49\times10^{-21}\ \text{J} + 1.12\times10^{-21}\ \text{J} = \boxed{-1.37\times10^{-21}\ \text{J}}$$

P7.66 $\mathcal{P}\Delta t = W = \Delta K = \dfrac{(\Delta m)v^2}{2}$

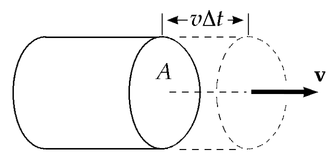

FIG. P7.66

The density is $\rho = \dfrac{\Delta m}{\text{vol}} = \dfrac{\Delta m}{A\Delta x}$.

Substituting this into the first equation and solving for $\mathcal{P}$, since $\dfrac{\Delta x}{\Delta t} = v$,

for a constant speed, we get $\boxed{\mathcal{P} = \dfrac{\rho A v^3}{2}}$.

Also, since $\mathcal{P} = Fv$, $\boxed{F = \dfrac{\rho A v^2}{2}}$.

Our model predicts the same proportionalities as the empirical equation, and gives $D = 1$ for the drag coefficient. Air actually slips around the moving object, instead of accumulating in front of it. For this reason, the drag coefficient is not necessarily unity. It is typically less than one for a streamlined object and can be greater than one if the airflow around the object is complicated.

P7.67 We evaluate $\displaystyle\int_{12.8}^{23.7} \frac{375dx}{x^3 + 3.75x}$ by calculating

$$\frac{375(0.100)}{(12.8)^3 + 3.75(12.8)} + \frac{375(0.100)}{(12.9)^3 + 3.75(12.9)} + \ldots \frac{375(0.100)}{(23.6)^3 + 3.75(23.6)} = 0.806$$

and

$$\frac{375(0.100)}{(12.9)^3 + 3.75(12.9)} + \frac{375(0.100)}{(13.0)^3 + 3.75(13.0)} + \ldots \frac{375(0.100)}{(23.7)^3 + 3.75(23.7)} = 0.791.$$

The answer must be between these two values. We may find it more precisely by using a value for Δx smaller than 0.100. Thus, we find the integral to be $\boxed{0.799\ \text{N}\cdot\text{m}}$.

***P7.68** $\mathscr{P} = \frac{1}{2} D\rho\pi r^2 v^3$

(a) $\mathscr{P}_a = \frac{1}{2} 1\left(1.20 \text{ kg/m}^3\right)\pi(1.5 \text{ m})^2(8 \text{ m/s})^3 = \boxed{2.17 \times 10^3 \text{ W}}$

(b) $\frac{\mathscr{P}_b}{\mathscr{P}_a} = \frac{v_b^3}{v_a^3} = \left(\frac{24 \text{ m/s}}{8 \text{ m/s}}\right)^3 = 3^3 = 27$

$\mathscr{P}_b = 27\left(2.17 \times 10^3 \text{ W}\right) = \boxed{5.86 \times 10^4 \text{ W}}$

P7.69 (a) The suggested equation $\mathscr{P}\Delta t = bwd$ implies all of the following cases:

(1) $\mathscr{P}\Delta t = b\left(\frac{w}{2}\right)(2d)$ (2) $\mathscr{P}\left(\frac{\Delta t}{2}\right) = b\left(\frac{w}{2}\right)d$

(3) $\mathscr{P}\left(\frac{\Delta t}{2}\right) = bw\left(\frac{d}{2}\right)$ and (4) $\left(\frac{\mathscr{P}}{2}\right)\Delta t = b\left(\frac{w}{2}\right)d$

These are all of the proportionalities Aristotle lists.

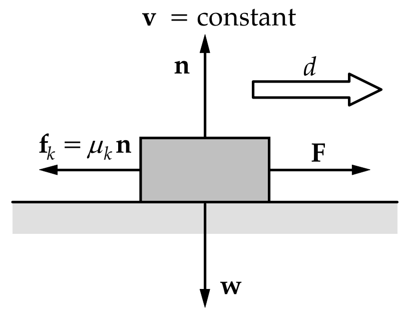

FIG. P7.69

(b) For one example, consider a horizontal force F pushing an object of weight w at constant velocity across a horizontal floor with which the object has coefficient of friction μ_k.

$\sum \mathbf{F} = m\mathbf{a}$ implies that:

$$+n - w = 0 \text{ and } F - \mu_k n = 0$$

so that $F = \mu_k w$

As the object moves a distance d, the agent exerting the force does work

$$W = Fd\cos\theta = Fd\cos 0° = \mu_k wd \text{ and puts out power } \mathscr{P} = \frac{W}{\Delta t}$$

This yields the equation $\mathscr{P}\Delta t = \mu_k wd$ which represents Aristotle's theory with $b = \mu_k$.

Our theory is more general than Aristotle's. Ours can also describe accelerated motion.

***P7.70** (a) So long as the spring force is greater than the friction force, the block will be gaining speed. The block slows down when the friction force becomes the greater. It has maximum speed when $kx_a - f_k = ma = 0$.

$\left(1.0 \times 10^3 \text{ N/m}\right)x_a - 4.0 \text{ N} = 0$ $\boxed{x = -4.0 \times 10^{-3} \text{ m}}$

(b) By the same logic,

$\left(1.0 \times 10^3 \text{ N/m}\right)x_b - 10.0 \text{ N} = 0$ $\boxed{x = -1.0 \times 10^{-2} \text{ m}}$

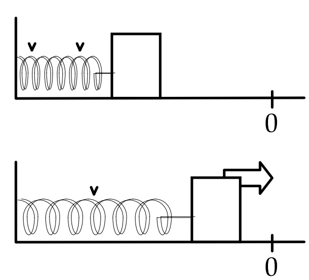

FIG. P7.70

ANSWERS TO EVEN PROBLEMS

P7.2 1.59×10^3 J

P7.4 (a) 3.28×10^{-2} J; (b) -3.28×10^{-2} J

P7.6 see the solution

P7.8 5.33 W

P7.10 16.0

P7.12 (a) see the solution; (b) −12.0 J

P7.14 50.0 J

P7.16 (a) 575 N/m; (b) 46.0 J

P7.18 (a) 9.00 kJ; (b) 11.7 kJ, larger by 29.6%

P7.20 (a) see the solution; (b) mgR

P7.22 (a) $\frac{mg}{k_1} + \frac{mg}{k_2}$; (b) $\left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$

P7.24 (a) 1.20 J; (b) 5.00 m/s; (c) 6.30 J

P7.26 (a) 60.0 J; (b) 60.0 J

P7.28 (a) 1.94 m/s; (b) 3.35 m/s; (c) 3.87 m/s

P7.30 (a) 3.78×10^{-16} J; (b) 1.35×10^{-14} N; (c) $1.48 \times 10^{+16}$ m/s^2; (d) 1.94 ns

P7.32 (a) 0.791 m/s; (b) 0.531 m/s

P7.34 (a) 329 J; (b) 0; (c) 0; (d) 185 J; (e) 144 J

P7.36 8.01 W

P7.38 $\sim 10^4$ W

P7.40 (a) 5.91 kW; (b) 11.1 kW

P7.42 No. (a) 582; (b) 90.5 W = 0.121 hp

P7.44 5.92 km/L

P7.46 90.0 J

P7.48 (a) $\cos\alpha = \frac{A_x}{A}$; $\cos\beta = \frac{A_y}{A}$; $\cos\gamma = \frac{A_z}{A}$; (b) see the solution

P7.50 (a) $a = \frac{40.1 \text{ kN}}{m^{1.8}}$; $b = 1.80$; (b) 294 J

P7.52 (a) $\frac{mgnhh_s}{v + nh_s}$; (b) $\frac{mgvh}{v + nh_s}$

P7.54 1.53×10^5 N upward

P7.56 see the solution

P7.58 (a) see the solution; (b) $2kL^2 + kA^2 - 2kL\sqrt{A^2 + L^2}$

P7.60 (a) $\mathbf{F}_1 = \left(20.5\hat{\mathbf{i}} + 14.3\hat{\mathbf{j}}\right)$ N; $\mathbf{F}_2 = \left(-36.4\hat{\mathbf{i}} + 21.0\hat{\mathbf{j}}\right)$ N; (b) $\left(-15.9\hat{\mathbf{i}} + 35.3\hat{\mathbf{j}}\right)$ N; (c) $\left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)$ m/s^2; (d) $\left(-5.54\hat{\mathbf{i}} + 23.7\hat{\mathbf{j}}\right)$ m/s; (e) $\left(-2.30\hat{\mathbf{i}} + 39.3\hat{\mathbf{j}}\right)$ m; (f) 1.48 kJ; (g) 1.48 kJ

P7.62 (a) see the solution; (b) 125 N/m ± 2%; (c) 13.1 N

P7.64 (a) 5.60 J; (b) 0.152; (c) 2.28 rev

P7.66 see the solution

P7.68 (a) 2.17 kW; (b) 58.6 kW

P7.70 (a) $x = -4.0$ mm; (b) −1.0 cm