8

Potential Energy

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q8.1 The final speed of the children will not depend on the slide length or the presence of bumps if there is no friction. If there is friction, a longer slide will result in a lower final speed. Bumps will have the same effect as they effectively lengthen the distance over which friction can do work, to decrease the total mechanical energy of the children.

Q8.2 Total energy is the sum of kinetic and potential energies. Potential energy can be negative, so the sum of kinetic plus potential can also be negative.

Q8.3 Both agree on the *change* in potential energy, and the kinetic energy. They may disagree on the value of gravitational potential energy, depending on their choice of a zero point.

Q8.4 (a) mgh is provided by the muscles.

(b) No further energy is supplied to the object-Earth system, but some chemical energy must be supplied to the muscles as they keep the weight aloft.

(c) The object loses energy mgh, giving it back to the muscles, where most of it becomes internal energy.

Q8.5 Lift a book from a low shelf to place it on a high shelf. The net change in its kinetic energy is zero, but the book-Earth system increases in gravitational potential energy. Stretch a rubber band to encompass the ends of a ruler. It increases in elastic energy. Rub your hands together or let a pearl drift down at constant speed in a bottle of shampoo. Each system (two hands; pearl and shampoo) increases in internal energy.

Q8.6 Three potential energy terms will appear in the expression of total mechanical energy, one for each conservative force. If you write an equation with initial energy on one side and final energy on the other, the equation contains six potential-energy terms.

Q8.7 (a) It does if it makes the object's speed change, but not if it only makes the direction of the velocity change.

(b) Yes, according to Newton's second law.

Q8.8 The original kinetic energy of the skidding can be degraded into kinetic energy of random molecular motion in the tires and the road: it is internal energy. If the brakes are used properly, the same energy appears as internal energy in the brake shoes and drums.

Q8.9 All the energy is supplied by foodstuffs that gained their energy from the sun.

Q8.10 Elastic potential energy of plates under stress plus gravitational energy is released when the plates "slip". It is carried away by mechanical waves.

Q8.11 The total energy of the ball-Earth system is conserved. Since the system initially has gravitational energy mgh and no kinetic energy, the ball will again have zero kinetic energy when it returns to its original position. Air resistance will cause the ball to come back to a point slightly below its initial position. On the other hand, if anyone gives a forward push to the ball anywhere along its path, the demonstrator will have to duck.

Q8.12 Using switchbacks requires no less work, as it does not change the *change* in potential energy from top to bottom. It does, however, require less force (of static friction on the rolling drive wheels of a car) to propel the car up the gentler slope. Less power is required if the work can be done over a longer period of time.

Q8.13 There is no work done since there is no change in kinetic energy. In this case, air resistance must be negligible since the acceleration is zero.

Q8.14 There is no violation. Choose the book as the system. You did work and the earth did work on the book. The average force you exerted just counterbalanced the weight of the book. The total work on the book is zero, and is equal to its overall change in kinetic energy.

Q8.15 Kinetic energy is greatest at the starting point. Gravitational energy is a maximum at the top of the flight of the ball.

Q8.16 Gravitational energy is proportional to mass, so it doubles.

Q8.17 In stirring cake batter and in weightlifting, your body returns to the same conformation after each stroke. During each stroke chemical energy is irreversibly converted into output work (and internal energy). This observation proves that muscular forces are nonconservative.

Q8.18 Let the gravitational energy be zero at the lowest point in the motion. If you start the vibration by pushing down on the block (2), its kinetic energy becomes extra elastic potential energy in the spring (U_s). After the block starts moving up at its lower turning point (3), this energy becomes both kinetic energy (*K*) and gravitational potential energy (U_g), and then just gravitational energy when the block is at its greatest height (1). The energy then turns back into kinetic and elastic potential energy, and the cycle repeats.

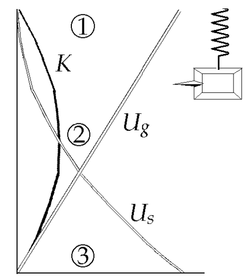

FIG. Q8.18

Q8.19 (a) Kinetic energy of the running athlete is transformed into elastic potential energy of the bent pole. This potential energy is transformed to a combination of kinetic energy and gravitational potential energy of the athlete and pole as the athlete approaches the bar. The energy is then all gravitational potential of the pole and the athlete as the athlete hopefully clears the bar. This potential energy then turns to kinetic energy as the athlete and pole fall to the ground. It immediately becomes internal energy as their macroscopic motion stops.

(b) Rotational kinetic energy of the athlete and shot is transformed into translational kinetic energy of the shot. As the shot goes through its trajectory as a projectile, the kinetic energy turns to a mix of kinetic and gravitational potential. The energy becomes internal energy as the shot comes to rest.

(c) Kinetic energy of the running athlete is transformed to a mix of kinetic and gravitational potential as the athlete becomes projectile going over a bar. This energy turns back into kinetic as the athlete falls down, and becomes internal energy as he stops on the ground.

The ultimate source of energy for all of these sports is the sun. See question 9.

Q8.20 Chemical energy in the fuel turns into internal energy as the fuel burns. Most of this leaves the car by heat through the walls of the engine and by matter transfer in the exhaust gases. Some leaves the system of fuel by work done to push down the piston. Of this work, a little results in internal energy in the bearings and gears, but most becomes work done on the air to push it aside. The work on the air immediately turns into internal energy in the air. If you use the windshield wipers, you take energy from the crankshaft and turn it into extra internal energy in the glass and wiper blades and wiper-motor coils. If you turn on the air conditioner, your end effect is to put extra energy out into the surroundings. You must apply the brakes at the end of your trip. As soon as the sound of the engine has died away, all you have to show for it is thermal pollution.

Q8.21 A graph of potential energy versus position is a straight horizontal line for a particle in neutral equilibrium. The graph represents a constant function.

Q8.22 The ball is in neutral equilibrium.

Q8.23 The ball is in stable equilibrium when it is directly below the pivot point. The ball is in unstable equilibrium when it is vertically above the pivot.

SOLUTIONS TO PROBLEMS

Section 8.1 **Potential Energy of a System**

P8.1 (a) With our choice for the zero level for potential energy when the car is at point B,

$$\boxed{U_B = 0}.$$

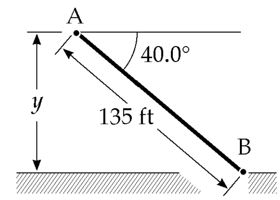

FIG. P8.1

When the car is at point A, the potential energy of the car-Earth system is given by

$$U_A = mgy$$

where y is the vertical height above zero level. With $135\text{ ft} = 41.1\text{ m}$, this height is found as:

$$y = (41.1\text{ m})\sin 40.0° = 26.4\text{ m}.$$

Thus,

$$U_A = (1\,000\text{ kg})(9.80\text{ m/s}^2)(26.4\text{ m}) = \boxed{2.59 \times 10^5\text{ J}}.$$

The change in potential energy as the car moves from A to B is

$$U_B - U_A = 0 - 2.59 \times 10^5\text{ J} = \boxed{-2.59 \times 10^5\text{ J}}.$$

(b) With our choice of the zero level when the car is at point A, we have $\boxed{U_A = 0}$. The potential energy when the car is at point B is given by $U_B = mgy$ where y is the vertical distance of point B below point A. In part (a), we found the magnitude of this distance to be 26.5 m. Because this distance is now below the zero reference level, it is a negative number.

Thus,

$$U_B = (1\,000\text{ kg})(9.80\text{ m/s}^2)(-26.5\text{ m}) = \boxed{-2.59 \times 10^5\text{ J}}.$$

The change in potential energy when the car moves from A to B is

$$U_B - U_A = -2.59 \times 10^5\text{ J} - 0 = \boxed{-2.59 \times 10^5\text{ J}}.$$

P8.2 (a) We take the zero configuration of system potential energy with the child at the lowest point of the arc. When the string is held horizontal initially, the initial position is 2.00 m above the zero level. Thus,

$$U_g = mgy = (400\text{ N})(2.00\text{ m}) = \boxed{800\text{ J}}.$$

(b) From the sketch, we see that at an angle of 30.0° the child is at a vertical height of $(2.00\text{ m})(1-\cos 30.0°)$ above the lowest point of the arc. Thus,

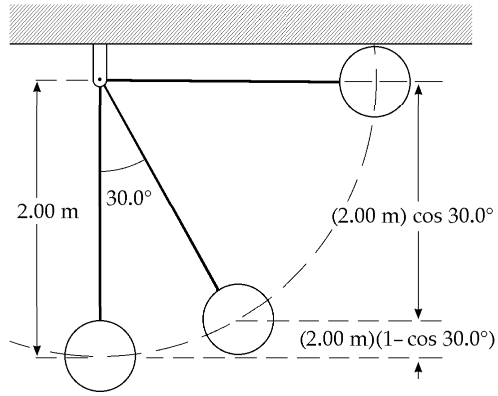

FIG. P8.2

$$U_g = mgy = (400\text{ N})(2.00\text{ m})(1-\cos 30.0°) = \boxed{107\text{ J}}.$$

(c) The zero level has been selected at the lowest point of the arc. Therefore, $\boxed{U_g = 0}$ at this location.

***P8.3** The volume flow rate is the volume of water going over the falls each second:

$$3\text{ m}(0.5\text{ m})(1.2\text{ m/s}) = 1.8\text{ m}^3/\text{s}$$

The mass flow rate is $\frac{m}{t} = \rho\frac{V}{t} = \left(1\,000\text{ kg/m}^3\right)\left(1.8\text{ m}^3/\text{s}\right) = 1\,800\text{ kg/s}$

If the stream has uniform width and depth, the speed of the water below the falls is the same as the speed above the falls. Then no kinetic energy, but only gravitational energy is available for conversion into internal and electric energy.

The input power is $\mathscr{P}_{\text{in}} = \frac{\text{energy}}{t} = \frac{mgy}{t} = \frac{m}{t}gy = (1\,800\text{ kg/s})\left(9.8\text{ m/s}^2\right)(5\text{ m}) = 8.82\times10^4\text{ J/s}$

The output power is $\mathscr{P}_{\text{useful}} = (\text{efficiency})\mathscr{P}_{\text{in}} = 0.25\left(8.82\times10^4\text{ W}\right) = \boxed{2.20\times10^4\text{ W}}$

The efficiency of electric generation at Hoover Dam is about 85%, with a head of water (vertical drop) of 174 m. Intensive research is underway to improve the efficiency of low head generators.

Section 8.2 The Isolated System—Conservation of Mechanical Energy

***P8.4** (a) One child in one jump converts chemical energy into mechanical energy in the amount that her body has as gravitational energy at the top of her jump:
$mgy = 36\text{ kg}\left(9.81\text{ m/s}^2\right)(0.25\text{ m}) = 88.3\text{ J}$. For all of the jumps of the children the energy is $12\left(1.05\times10^6\right)88.3\text{ J} = \boxed{1.11\times10^9\text{ J}}$.

(b) The seismic energy is modeled as $E = \frac{0.01}{100}1.11\times10^9\text{ J} = 1.11\times10^5\text{ J}$, making the Richter magnitude $\frac{\log E - 4.8}{1.5} = \frac{\log 1.11\times10^5 - 4.8}{1.5} = \frac{5.05-4.8}{1.5} = \boxed{0.2}$.

P8.5 $U_i + K_i = U_f + K_f$: $mgh + 0 = mg(2R) + \frac{1}{2}mv^2$

$$g(3.50R) = 2g(R) + \frac{1}{2}v^2$$

$$\boxed{v = \sqrt{3.00gR}}$$

$\sum F = m\frac{v^2}{R}$: $n + mg = m\frac{v^2}{R}$

$$n = m\left[\frac{v^2}{R} - g\right] = m\left[\frac{3.00gR}{R} - g\right] = 2.00mg$$

$$n = 2.00\left(5.00\times10^{-3}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)$$

$$= \boxed{0.098\,0\text{ N downward}}$$

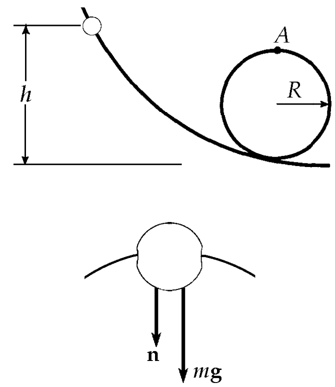

FIG. P8.5

P8.6 From leaving ground to the highest point, $K_i + U_i = K_f + U_f$

$$\frac{1}{2}m(6.00\text{ m/s})^2 + 0 = 0 + m\left(9.80\text{ m/s}^2\right)y$$

The mass makes no difference: $\therefore y = \frac{(6.00\text{ m/s})^2}{(2)\left(9.80\text{ m/s}^2\right)} = \boxed{1.84\text{ m}}$

***P8.7** (a) $\frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2$

$$0 + \frac{1}{2}(10\text{ N/m})(-0.18\text{ m})^2 = \frac{1}{2}(0.15\text{ kg})v_f^2 + 0$$

$$v_f = (0.18\text{ m})\sqrt{\left(\frac{10\text{ N}}{0.15\text{ kg}\cdot\text{m}}\right)\left(\frac{1\text{ kg}\cdot\text{m}}{1\text{ N}\cdot\text{s}^2}\right)} = \boxed{1.47\text{ m/s}}$$

(b) $K_i + U_{si} = K_f + U_{sf}$

$$0 + \frac{1}{2}(10\text{ N/m})(-0.18\text{ m})^2 = \frac{1}{2}(0.15\text{ kg})v_f^2 + \frac{1}{2}(10\text{ N/m})(0.25\text{ m} - 0.18\text{ m})^2$$

$$0.162\text{ J} = \frac{1}{2}(0.15\text{ kg})v_f^2 + 0.024\,5\text{ J}$$

$$v_f = \sqrt{\frac{2(0.138\text{ J})}{0.15\text{ kg}}} = \boxed{1.35\text{ m/s}}$$

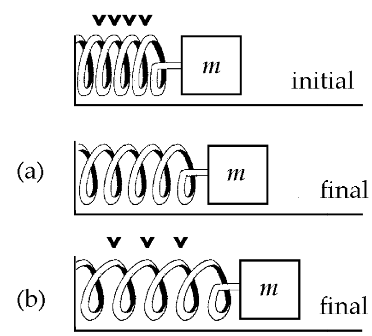

FIG. P8.7

***P8.8** The energy of the car is $E = \frac{1}{2}mv^2 + mgy$

$E = \frac{1}{2}mv^2 + mgd\sin\theta$ where d is the distance it has moved along the track.

$\mathcal{P} = \frac{dE}{dt} = mv\frac{dv}{dt} + mgv\sin\theta$

(a) When speed is constant,

$$\mathcal{P} = mgv\sin\theta = 950\text{ kg}\left(9.80\text{ m/s}^2\right)(2.20\text{ m/s})\sin 30° = \boxed{1.02\times 10^4\text{ W}}$$

(b) $\frac{dv}{dt} = a = \frac{2.2\text{ m/s} - 0}{12\text{ s}} = 0.183\text{ m/s}^2$

Maximum power is injected just before maximum speed is attained:

$$\mathcal{P} = mva + mgv\sin\theta = 950\text{ kg}(2.2\text{ m/s})\left(0.183\text{ m/s}^2\right) + 1.02\times 10^4\text{ W} = \boxed{1.06\times 10^4\text{ W}}$$

(c) At the top end,

$$\frac{1}{2}mv^2 + mgd\sin\theta = 950\text{ kg}\left(\frac{1}{2}(2.20\text{ m/s})^2 + \left(9.80\text{ m/s}^2\right)1\,250\text{ m}\sin 30°\right) = \boxed{5.82\times 10^6\text{ J}}$$

***P8.9** (a) Energy of the object-Earth system is conserved as the object moves between the release point and the lowest point. We choose to measure heights from $y = 0$ at the top end of the string.

$$\left(K + U_g\right)_i = \left(K + U_g\right)_f: \qquad 0 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

$$\left(9.8\text{ m/s}^2\right)(-2\text{ m}\cos 30°) = \frac{1}{2}v_f^2 + \left(9.8\text{ m/s}^2\right)(-2\text{ m})$$

$$v_f = \sqrt{2\left(9.8\text{ m/s}^2\right)(2\text{ m})(1 - \cos 30°)} = \boxed{2.29\text{ m/s}}$$

(b) Choose the initial point at $\theta = 30°$ and the final point at $\theta = 15°$:

$$0 + mg(-L\cos 30°) = \frac{1}{2}mv_f^2 + mg(-L\cos 15°)$$

$$v_f = \sqrt{2gL(\cos 15° - \cos 30°)} = \sqrt{2\left(9.8\text{ m/s}^2\right)(2\text{ m})(\cos 15° - \cos 30°)} = \boxed{1.98\text{ m/s}}$$

P8.10 Choose the zero point of gravitational potential energy of the object-spring-Earth system as the configuration in which the object comes to rest. Then because the incline is frictionless, we have $E_B = E_A$:

$$K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$$

or

$$0 + mg(d + x)\sin\theta + 0 = 0 + 0 + \frac{1}{2}kx^2.$$

Solving for d gives

$$d = \boxed{\frac{kx^2}{2mg\sin\theta} - x}.$$

P8.11 From conservation of energy for the block-spring-Earth system,

$$U_{gt} = U_{si},$$

or

$$(0.250 \text{ kg})\left(9.80 \text{ m/s}^2\right)h = \left(\frac{1}{2}\right)(5\,000 \text{ N/m})(0.100 \text{ m})^2$$

This gives a maximum height $h = \boxed{10.2 \text{ m}}$.

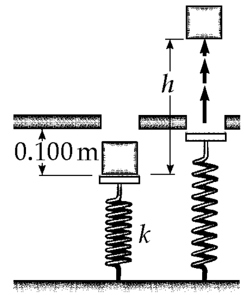

FIG. P8.11

P8.12 (a) The force needed to hang on is equal to the force F the trapeze bar exerts on the performer.

From the free-body diagram for the performer's body, as shown,

$$F - mg\cos\theta = m\frac{v^2}{\ell}$$

or

$$F = mg\cos\theta + m\frac{v^2}{\ell}$$

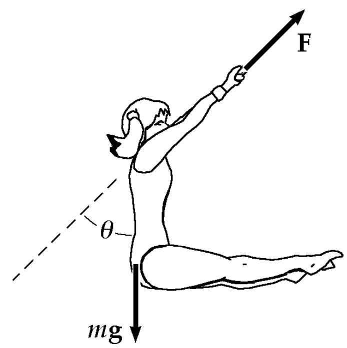

FIG. P8.12

Apply conservation of mechanical energy of the performer-Earth system as the performer moves between the starting point and any later point:

$$mg(\ell - \ell\cos\theta_i) = mg(\ell - \ell\cos\theta) + \frac{1}{2}mv^2$$

Solve for $\frac{mv^2}{\ell}$ and substitute into the force equation to obtain $F = \boxed{mg(3\cos\theta - 2\cos\theta_i)}$.

(b) At the bottom of the swing, $\theta = 0°$ so

$$F = mg(3 - 2\cos\theta_i)$$
$$F = 2mg = mg(3 - 2\cos\theta_i)$$

which gives

$$\theta_i = \boxed{60.0°}.$$

P8.13 Using conservation of energy for the system of the Earth and the two objects

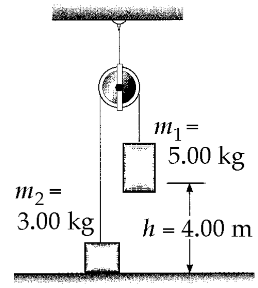

FIG. P8.13

(a) $$(5.00\text{ kg})g(4.00\text{ m}) = (3.00\text{ kg})g(4.00\text{ m}) + \frac{1}{2}(5.00+3.00)v^2$$

$$v = \sqrt{19.6} = \boxed{4.43\text{ m/s}}$$

(b) Now we apply conservation of energy for the system of the 3.00 kg object and the Earth during the time interval between the instant when the string goes slack and the instant at which the 3.00 kg object reaches its highest position in its free fall.

$$\frac{1}{2}(3.00)v^2 = mg\,\Delta y = 3.00g\Delta y$$

$$\Delta y = 1.00\text{ m}$$

$$y_{\max} = 4.00\text{ m} + \Delta y = \boxed{5.00\text{ m}}$$

P8.14 $m_1 > m_2$

(a) $$m_1 gh = \frac{1}{2}(m_1 + m_2)v^2 + m_2 gh$$

$$v = \boxed{\sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)}}}$$

(b) Since m_2 has kinetic energy $\frac{1}{2}m_2 v^2$, it will rise an additional height Δh determined from

$$m_2 g\,\Delta h = \frac{1}{2}m_2 v^2$$

or from (a),

$$\Delta h = \frac{v^2}{2g} = \frac{(m_1 - m_2)h}{(m_1 + m_2)}$$

The total height m_2 reaches is $h + \Delta h = \boxed{\dfrac{2m_1 h}{m_1 + m_2}}$.

P8.15 The force of tension and subsequent force of compression in the rod do no work on the ball, since they are perpendicular to each step of displacement. Consider energy conservation of the ball-Earth system between the instant just after you strike the ball and the instant when it reaches the top. The speed at the top is zero if you hit it just hard enough to get it there.

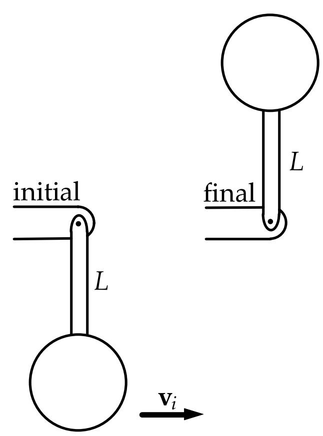

FIG. P8.15

$K_i + U_{gi} = K_f + U_{gf}$: $$\frac{1}{2}mv_i^2 + 0 = 0 + mg(2L)$$

$$v_i = \sqrt{4gL} = \sqrt{4(9.80)(0.770)}$$

$$v_i = \boxed{5.49\text{ m/s}}$$

***P8.16** efficiency $= \dfrac{\text{useful output energy}}{\text{total input energy}} = \dfrac{\text{useful output power}}{\text{total input power}}$

$$e = \frac{m_{\text{water}} gy/t}{(1/2)m_{\text{air}}\left(v^2/t\right)} = \frac{2\rho_{\text{water}}\left(v_{\text{water}}/t\right)gy}{\rho_{\text{air}}\pi r^2\left(\ell v^2/t\right)} = \frac{2\rho_{\text{w}}\left(v_{\text{w}}/t\right)gy}{\rho_{\text{a}}\pi r^2 v^3}$$

where ℓ is the length of a cylinder of air passing through the mill and v_{w} is the volume of water pumped in time t. We need inject negligible kinetic energy into the water because it starts and ends at rest.

$$\frac{v_{\text{w}}}{t} = \frac{e\rho_{\text{a}}\pi r^2 v^3}{2\rho_{\text{w}} gy} = \frac{0.275\left(1.20\ \text{kg/m}^3\right)\pi(1.15\ \text{m})^2(11\ \text{m/s})^3}{2\left(1\,000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)35\ \text{m}}$$

$$= 2.66\times 10^{-3}\ \text{m}^3/\text{s}\left(\frac{1\,000\ \text{L}}{1\ \text{m}^3}\right)\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = \boxed{160\ \text{L/min}}$$

P8.17 (a) $K_i + U_{gi} = K_f + U_{gf}$

$$\frac{1}{2}mv_i^2 + 0 = \frac{1}{2}mv_f^2 + mgy_f$$

$$\frac{1}{2}mv_{xi}^2 + \frac{1}{2}mv_{yi}^2 = \frac{1}{2}mv_{xf}^2 + mgy_f$$

But $v_{xi} = v_{xf}$, so for the first ball

$$y_f = \frac{v_{yi}^2}{2g} = \frac{(1\,000\sin 37.0°)^2}{2(9.80)} = \boxed{1.85\times 10^4\ \text{m}}$$

and for the second

$$y_f = \frac{(1\,000)^2}{2(9.80)} = \boxed{5.10\times 10^4\ \text{m}}$$

(b) The total energy of each is constant with value

$$\frac{1}{2}(20.0\ \text{kg})(1\,000\ \text{m/s})^2 = \boxed{1.00\times 10^7\ \text{J}}.$$

P8.18 In the swing down to the breaking point, energy is conserved:

$$mgr\cos\theta = \frac{1}{2}mv^2$$

at the breaking point consider radial forces

$$\sum F_r = ma_r$$

$$+T_{\max} - mg\cos\theta = m\frac{v^2}{r}$$

Eliminate $\frac{v^2}{r} = 2g\cos\theta$

$$T_{\max} - mg\cos\theta = 2mg\cos\theta$$

$$T_{\max} = 3mg\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{T_{\max}}{3mg}\right) = \cos^{-1}\left(\frac{44.5\text{ N}}{3(2.00\text{ kg})(9.80\text{ m/s}^2)}\right)$$

$$\theta = \boxed{40.8°}$$

***P8.19** (a) For a 5-m cord the spring constant is described by $F = kx$, $mg = k(1.5\text{ m})$. For a longer cord of length L the stretch distance is longer so the spring constant is smaller in inverse proportion:

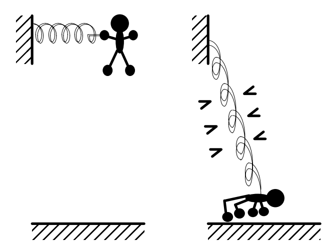

FIG. P8.19(a)

$$k = \frac{5\text{ m}}{L}\frac{mg}{1.5\text{ m}} = 3.33\,mg/L$$

$$(K + U_g + U_s)_i = (K + U_g + U_s)_f$$

$$0 + mgy_i + 0 = 0 + mgy_f + \frac{1}{2}kx_f^2$$

$$mg(y_i - y_f) = \frac{1}{2}kx_f^2 = \frac{1}{2}3.33\frac{mg}{L}x_f^2$$

here $y_i - y_f = 55\text{ m} = L + x_f$

$$55.0\text{ m}L = \frac{1}{2}3.33(55.0\text{ m} - L)^2$$

$$55.0\text{ m}L = 5.04\times10^3\text{ m}^2 - 183\text{ m}L + 1.67L^2$$

$$0 = 1.67L^2 - 238L + 5.04\times10^3 = 0$$

$$L = \frac{238 \pm \sqrt{238^2 - 4(1.67)(5.04\times10^3)}}{2(1.67)} = \frac{238\pm152}{3.33} = \boxed{25.8\text{ m}}$$

only the value of L less than 55 m is physical.

(b) $k = 3.33\frac{mg}{25.8\text{ m}}$ $\quad x_{\max} = x_f = 55.0\text{ m} - 25.8\text{ m} = 29.2\text{ m}$

$\sum F = ma$ $\quad +kx_{\max} - mg = ma$

$$3.33\frac{mg}{25.8\text{ m}}29.2\text{ m} - mg = ma$$

$$a = 2.77g = \boxed{27.1\text{ m/s}^2}$$

***P8.20** When block B moves up by 1 cm, block A moves down by 2 cm and the separation becomes 3 cm. We then choose the final point to be when B has moved up by $\frac{h}{3}$ and has speed $\frac{v_A}{2}$. Then A has moved down $\frac{2h}{3}$ and has speed v_A:

$$\left(K_A + K_B + U_g\right)_i = \left(K_A + K_B + U_g\right)_f$$

$$0+0+0 = \frac{1}{2}mv_A^2 + \frac{1}{2}m\left(\frac{v_A}{2}\right)^2 + \frac{mgh}{3} - \frac{mg2h}{3}$$

$$\frac{mgh}{3} = \frac{5}{8}mv_A^2$$

$$\boxed{v_A = \sqrt{\frac{8gh}{15}}}$$

Section 8.3 Conservative and Nonconservative Forces

P8.21 $F_g = mg = (4.00 \text{ kg})(9.80 \text{ m/s}^2) = 39.2 \text{ N}$

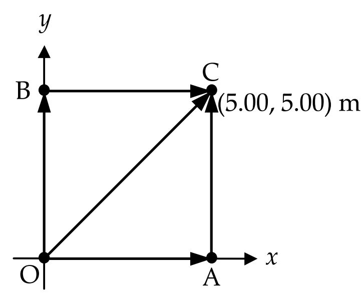

FIG. P8.21

(a) Work along OAC = work along OA + work along AC

$= F_g(\text{OA})\cos 90.0° + F_g(\text{AC})\cos 180°$

$= (39.2 \text{ N})(5.00 \text{ m}) + (39.2 \text{ N})(5.00 \text{ m})(-1)$

$= \boxed{-196 \text{ J}}$

(b) W along OBC = W along OB + W along BC

$= (39.2 \text{ N})(5.00 \text{ m})\cos 180° + (39.2 \text{ N})(5.00 \text{ m})\cos 90.0°$

$= \boxed{-196 \text{ J}}$

(c) Work along OC $= F_g(\text{OC})\cos 135°$

$= (39.2 \text{ N})\left(5.00 \times \sqrt{2} \text{ m}\right)\left(-\frac{1}{\sqrt{2}}\right) = \boxed{-196 \text{ J}}$

The results should all be the same, since gravitational forces are conservative.

P8.22 (a) $W = \int \mathbf{F} \cdot d\mathbf{r}$ and if the force is constant, this can be written as

$W = \mathbf{F} \cdot \int d\mathbf{r} = \boxed{\mathbf{F} \cdot \left(\mathbf{r}_f - \mathbf{r}_i\right), \text{ which depends only on end points, not path.}}$

(b) $W = \int \mathbf{F} \cdot d\mathbf{r} = \int \left(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}\right) \cdot \left(dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}}\right) = (3.00 \text{ N}) \int_0^{5.00 \text{ m}} dx + (4.00 \text{ N}) \int_0^{5.00 \text{ m}} dy$

$W = (3.00 \text{ N})x\Big|_0^{5.00 \text{ m}} + (4.00 \text{ N})y\Big|_0^{5.00 \text{ m}} = 15.0 \text{ J} + 20.0 \text{ J} = \boxed{35.0 \text{ J}}$

The same calculation applies for all paths.

P8.23 (a) $W_{OA} = \int_0^{5.00\text{ m}} dx\hat{\mathbf{i}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right) = \int_0^{5.00\text{ m}} 2y\,dx$

and since along this path, $y = 0$ $\quad W_{OA} = 0$

$$W_{AC} = \int_0^{5.00\text{ m}} dy\hat{\mathbf{j}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right) = \int_0^{5.00\text{ m}} x^2\,dy$$

For $x = 5.00$ m, $\quad W_{AC} = 125$ J

and $\quad W_{OAC} = 0 + 125 = \boxed{125\text{ J}}$

(b) $W_{OB} = \int_0^{5.00\text{ m}} dy\hat{\mathbf{j}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right) = \int_0^{5.00\text{ m}} x^2\,dy$

since along this path, $x = 0$, $\quad W_{OB} = 0$

$$W_{BC} = \int_0^{5.00\text{ m}} dx\hat{\mathbf{i}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right) = \int_0^{5.00\text{ m}} 2y\,dx$$

since $y = 5.00$ m, $\quad W_{BC} = 50.0$ J

$$W_{OBC} = 0 + 50.0 = \boxed{50.0\text{ J}}$$

(c) $W_{OC} = \int\left(dx\hat{\mathbf{i}}+dy\hat{\mathbf{j}}\right)\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right) = \int\left(2y\,dx + x^2\,dy\right)$

Since $x = y$ along OC, $\quad W_{OC} = \int_0^{5.00\text{ m}}\left(2x + x^2\right)dx = \boxed{66.7\text{ J}}$

(d) F is $\boxed{\text{nonconservative}}$ since the work done is path dependent.

P8.24 (a) $(\Delta K)_{A\to B} = \sum W = W_g = mg\Delta h = mg(5.00 - 3.20)$

$$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = m(9.80)(1.80)$$

$$v_B = \boxed{5.94\ \text{m/s}}$$

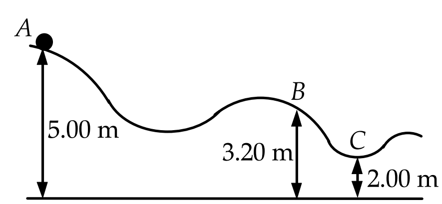

FIG. P8.24

Similarly, $v_C = \sqrt{v_A^2 + 2g(5.00 - 2.00)} = \boxed{7.67\ \text{m/s}}$

(b) $W_g\big|_{A\to C} = mg(3.00\text{ m}) = \boxed{147\text{ J}}$

P8.25 (a) $\mathbf{F} = \left(3.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}\right)$ N

$m = 4.00$ kg

$\mathbf{r} = \left(2.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}}\right)$ m

$W = 3.00(2.00) + 5.00(-3.00) = \boxed{-9.00\text{ J}}$

The result does not depend on the path since the force is conservative.

(b) $W = \Delta K$

$$-9.00 = \frac{4.00v^2}{2} - 4.00\left(\frac{(4.00)^2}{2}\right)$$

so $v = \sqrt{\frac{32.0 - 9.00}{2.00}} = \boxed{3.39\text{ m/s}}$

(c) $\Delta U = -W = \boxed{9.00\text{ J}}$

Section 8.4 **Changes in Mechanical Energy for Nonconservative Forces**

P8.26 (a) $U_f = K_i - K_f + U_i$ $\qquad U_f = 30.0 - 18.0 + 10.0 = \boxed{22.0\text{ J}}$

$\boxed{E = 40.0\text{ J}}$

(b) Yes, $\Delta E_{\text{mech}} = \Delta K + \Delta U$ is not equal to zero. For conservative forces $\Delta K + \Delta U = 0$.

P8.27 The distance traveled by the ball from the top of the arc to the bottom is πR. The work done by the non-conservative force, the force exerted by the pitcher,

is $\qquad \Delta E = F\Delta r\cos 0° = F(\pi R)$.

We shall assign the gravitational energy of the ball-Earth system to be zero with the ball at the bottom of the arc.

Then $\qquad \Delta E_{\text{mech}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgy_f - mgy_i$

becomes $\qquad \frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + mgy_i + F(\pi R)$

or $\qquad v_f = \sqrt{v_i^2 + 2gy_i + \frac{2F(\pi R)}{m}} = \sqrt{(15.0)^2 + 2(9.80)(1.20) + \frac{2(30.0)\pi(0.600)}{0.250}}$

$v_f = \boxed{26.5\text{ m/s}}$

***P8.28** The useful output energy is

$$120\text{ Wh}(1 - 0.60) = mg\left(y_f - y_i\right) = F_g\Delta y$$

$$\Delta y = \frac{120\text{ W}(3\,600\text{ s})0.40}{890\text{ N}}\left(\frac{\text{J}}{\text{W}\cdot\text{s}}\right)\left(\frac{\text{N}\cdot\text{m}}{\text{J}}\right) = \boxed{194\text{ m}}$$

***P8.29** As the locomotive moves up the hill at constant speed, its output power goes into internal energy plus gravitational energy of the locomotive-Earth system:

$$\mathcal{P}t = mgy + f\Delta r = mg\Delta r\sin\theta + f\Delta r \qquad \mathcal{P} = mgv_f\sin\theta + fv_f$$

As the locomotive moves on level track,

$$\mathcal{P} = fv_i \qquad 1\,000\text{ hp}\left(\frac{746\text{ W}}{1\text{ hp}}\right) = f(27\text{ m/s}) \qquad f = 2.76\times10^4\text{ N}$$

Then also $746\,000\text{ W} = (160\,000\text{ kg})(9.8\text{ m/s}^2)v_f\left(\frac{5\text{ m}}{100\text{ m}}\right) + (2.76\times10^4\text{ N})v_f$

$$v_f = \frac{746\,000\text{ W}}{1.06\times10^5\text{ N}} = \boxed{7.04\text{ m/s}}$$

P8.30 We shall take the zero level of gravitational potential energy to be at the lowest level reached by the diver under the water, and consider the energy change from when the diver started to fall until he came to rest.

$$\Delta E = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgy_f - mgy_i = f_k d\cos180°$$

$$0 - 0 - mg(y_i - y_f) = -f_k d$$

$$f_k = \frac{mg(y_i - y_f)}{d} = \frac{(70.0\text{ kg})(9.80\text{ m/s}^2)(10.0\text{ m} + 5.00\text{ m})}{5.00\text{ m}} = \boxed{2.06\text{ kN}}$$

P8.31 $U_i + K_i + \Delta E_{\text{mech}} = U_f + K_f$: $\quad m_2gh - fh = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2$

$$f = \mu n = \mu m_1 g$$

$$m_2gh - \mu m_1 gh = \frac{1}{2}(m_1 + m_2)v^2$$

$$v^2 = \frac{2(m_2 - \mu m_1)(hg)}{m_1 + m_2}$$

$$v = \sqrt{\frac{2(9.80\text{ m/s}^2)(1.50\text{ m})[5.00\text{ kg} - 0.400(3.00\text{ kg})]}{8.00\text{ kg}}} = \boxed{3.74\text{ m/s}}$$

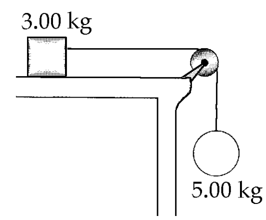

FIG. P8.31

P8.32 $\Delta E_{\text{mech}} = (K_f - K_i) + (U_{gf} - U_{gi})$

But $\Delta E_{\text{mech}} = W_{\text{app}} - f\Delta x$, where W_{app} is the work the boy did pushing forward on the wheels.

Thus, $\quad W_{\text{app}} = (K_f - K_i) + (U_{gf} - U_{gi}) + f\Delta x$

or $\quad W_{\text{app}} = \frac{1}{2}m(v_f^2 - v_i^2) + mg(-h) + f\Delta x$

$$W_{\text{app}} = \frac{1}{2}(47.0)\left[(6.20)^2 - (1.40)^2\right] - (47.0)(9.80)(2.60) + (41.0)(12.4)$$

$$W_{\text{app}} = \boxed{168\text{ J}}$$

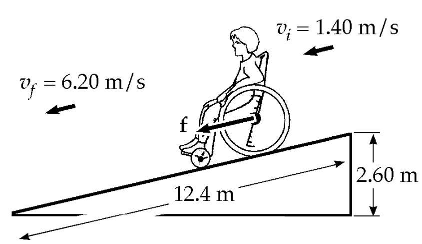

FIG. P8.32

P8.33 (a) $\Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = -\frac{1}{2}mv_i^2 = \boxed{-160\text{ J}}$

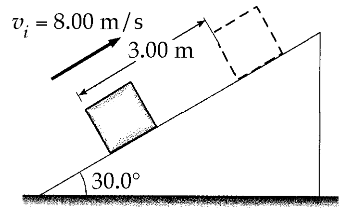

FIG. P8.33

(b) $\Delta U = mg(3.00\text{ m})\sin 30.0° = \boxed{73.5\text{ J}}$

(c) The mechanical energy converted due to friction is 86.5 J

$$f = \frac{86.5\text{ J}}{3.00\text{ m}} = \boxed{28.8\text{ N}}$$

(d) $f = \mu_k n = \mu_k mg\cos 30.0° = 28.8\text{ N}$

$$\mu_k = \frac{28.8\text{ N}}{(5.00\text{ kg})\left(9.80\text{ m/s}^2\right)\cos 30.0°} = \boxed{0.679}$$

P8.34 Consider the whole motion: $K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$

(a) $0 + mgy_i - f_1\Delta x_1 - f_2\Delta x_2 = \frac{1}{2}mv_f^2 + 0$

$$(80.0\text{ kg})\left(9.80\text{ m/s}^2\right)1\,000\text{ m} - (50.0\text{ N})(800\text{ m}) - (3\,600\text{ N})(200\text{ m}) = \frac{1}{2}(80.0\text{ kg})v_f^2$$

$$784\,000\text{ J} - 40\,000\text{ J} - 720\,000\text{ J} = \frac{1}{2}(80.0\text{ kg})v_f^2$$

$$v_f = \sqrt{\frac{2(24\,000\text{ J})}{80.0\text{ kg}}} = \boxed{24.5\text{ m/s}}$$

(b) $\boxed{\text{Yes}}$ this is too fast for safety.

(c) Now in the same energy equation as in part (a), Δx_2 is unknown, and $\Delta x_1 = 1\,000\text{ m} - \Delta x_2$:

$$784\,000\text{ J} - (50.0\text{ N})(1\,000\text{ m} - \Delta x_2) - (3\,600\text{ N})\Delta x_2 = \frac{1}{2}(80.0\text{ kg})(5.00\text{ m/s})^2$$

$$784\,000\text{ J} - 50\,000\text{ J} - (3\,550\text{ N})\Delta x_2 = 1\,000\text{ J}$$

$$\Delta x_2 = \frac{733\,000\text{ J}}{3\,550\text{ N}} = \boxed{206\text{ m}}$$

(d) Really the air drag will depend on the skydiver's speed. It will be larger than her 784 N weight only after the chute is opened. It will be nearly equal to 784 N before she opens the chute and again before she touches down, whenever she moves near terminal speed.

P8.35 (a) $(K+U)_i + \Delta E_{\text{mech}} = (K+U)_f$:

$$0 + \frac{1}{2}kx^2 - f\Delta x = \frac{1}{2}mv^2 + 0$$

$$\frac{1}{2}(8.00\text{ N/m})\left(5.00\times10^{-2}\text{ m}\right)^2 - \left(3.20\times10^{-2}\text{ N}\right)(0.150\text{ m}) = \frac{1}{2}\left(5.30\times10^{-3}\text{ kg}\right)v^2$$

$$v = \sqrt{\frac{2\left(5.20\times10^{-3}\text{ J}\right)}{5.30\times10^{-3}\text{ kg}}} = \boxed{1.40\text{ m/s}}$$

(b) When the spring force just equals the friction force, the ball will stop speeding up. Here $\left|\mathbf{F}_s\right| = kx$; the spring is compressed by

$$\frac{3.20\times10^{-2}\text{ N}}{8.00\text{ N/m}} = 0.400\text{ cm}$$

and the ball has moved

$$5.00\text{ cm} - 0.400\text{ cm} = \boxed{4.60\text{ cm from the start.}}$$

(c) Between start and maximum speed points,

$$\frac{1}{2}kx_i^2 - f\Delta x = \frac{1}{2}mv^2 + \frac{1}{2}kx_f^2$$

$$\frac{1}{2}8.00\left(5.00\times10^{-2}\right)^2 - \left(3.20\times10^{-2}\right)\left(4.60\times10^{-2}\right) = \frac{1}{2}\left(5.30\times10^{-3}\right)v^2 + \frac{1}{2}8.00\left(4.00\times10^{-3}\right)^2$$

$$v = \boxed{1.79\text{ m/s}}$$

P8.36 $\sum F_y = n - mg\cos 37.0° = 0$

$\therefore n = mg\cos 37.0° = 400\text{ N}$

$f = \mu n = 0.250(400\text{ N}) = 100\text{ N}$

$-f\Delta x = \Delta E_{\text{mech}}$

$(-100)(20.0) = \Delta U_A + \Delta U_B + \Delta K_A + \Delta K_B$

$\Delta U_A = m_A g\left(h_f - h_i\right) = (50.0)(9.80)(20.0\sin 37.0°) = 5.90\times10^3$

$\Delta U_B = m_B g\left(h_f - h_i\right) = (100)(9.80)(-20.0) = -1.96\times10^4$

$\Delta K_A = \frac{1}{2}m_A\left(v_f^2 - v_i^2\right)$

$\Delta K_B = \frac{1}{2}m_B\left(v_f^2 - v_i^2\right) = \frac{m_B}{m_A}\Delta K_A = 2\Delta K_A$

Adding and solving, $\Delta K_A = \boxed{3.92\text{ kJ}}$.

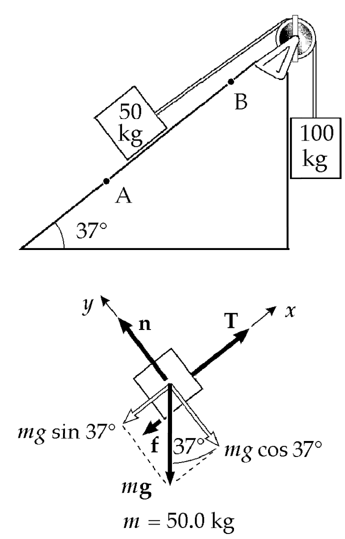

FIG. P8.36

P8.37 (a) The object moved down distance $1.20\text{ m}+x$. Choose $y=0$ at its lower point.

$$K_i+U_{gi}+U_{si}+\Delta E_{\text{mech}}=K_f+U_{gf}+U_{sf}$$
$$0+mgy_i+0+0=0+0+\frac{1}{2}kx^2$$
$$(1.50\text{ kg})\left(9.80\text{ m/s}^2\right)(1.20\text{ m}+x)=\frac{1}{2}(320\text{ N/m})x^2$$
$$0=(160\text{ N/m})x^2-(14.7\text{ N})x-17.6\text{ J}$$
$$x=\frac{14.7\text{ N}\pm\sqrt{(-14.7\text{ N})^2-4(160\text{ N/m})(-17.6\text{ N}\cdot\text{m})}}{2(160\text{ N/m})}$$
$$x=\frac{14.7\text{ N}\pm 107\text{ N}}{320\text{ N/m}}$$

The negative root tells how high the object will rebound if it is instantly glued to the spring. We want

$$x=\boxed{0.381\text{ m}}$$

(b) From the same equation,

$$(1.50\text{ kg})\left(1.63\text{ m/s}^2\right)(1.20\text{ m}+x)=\frac{1}{2}(320\text{ N/m})x^2$$
$$0=160x^2-2.44x-2.93$$

The positive root is $x=\boxed{0.143\text{ m}}$.

(c) The equation expressing the energy version of the nonisolated system model has one more term:

$$mgy_i-f\Delta x=\frac{1}{2}kx^2$$
$$(1.50\text{ kg})\left(9.80\text{ m/s}^2\right)(1.20\text{ m}+x)-0.700\text{ N}(1.20\text{ m}+x)=\frac{1}{2}(320\text{ N/m})x^2$$
$$17.6\text{ J}+14.7\text{ N}x-0.840\text{ J}-0.700\text{ N}x=160\text{ N/m}\,x^2$$
$$160x^2-14.0x-16.8=0$$
$$x=\frac{14.0\pm\sqrt{(14.0)^2-4(160)(-16.8)}}{320}$$
$$x=\boxed{0.371\text{ m}}$$

P8.38 The total mechanical energy of the skysurfer-Earth system is

$$E_{\text{mech}} = K + U_g = \frac{1}{2}mv^2 + mgh.$$

Since the skysurfer has constant speed,

$$\frac{dE_{\text{mech}}}{dt} = mv\frac{dv}{dt} + mg\frac{dh}{dt} = 0 + mg(-v) = -mgv.$$

The rate the system is losing mechanical energy is then

$$\left|\frac{dE_{\text{mech}}}{dt}\right| = mgv = (75.0\text{ kg})\left(9.80\text{ m/s}^2\right)(60.0\text{ m/s}) = \boxed{44.1\text{ kW}}.$$

***P8.39** (a) Let m be the mass of the whole board. The portion on the rough surface has mass $\frac{mx}{L}$. The normal force supporting it is $\frac{mxg}{L}$ and the frictional force is $\frac{\mu_k mgx}{L} = ma$. Then $\boxed{a = \frac{\mu_k gx}{L}\text{ opposite to the motion.}}$

(b) In an incremental bit of forward motion dx, the kinetic energy converted into internal energy is $f_k dx = \frac{\mu_k mgx}{L}dx$. The whole energy converted is

$$\frac{1}{2}mv^2 = \int_0^L \frac{\mu_k mgx}{L}dx = \frac{\mu_k mg}{L}\frac{x^2}{2}\bigg|_0^L = \frac{\mu_k mgL}{2}$$

$$\boxed{v = \sqrt{\mu_k gL}}$$

Section 8.5 **Relationship Between Conservative Forces and Potential Energy**

P8.40 (a) $U = -\int_0^x \left(-Ax + Bx^2\right)dx = \boxed{\frac{Ax^2}{2} - \frac{Bx^3}{3}}$

(b) $\Delta U = -\int_{2.00\text{ m}}^{3.00\text{ m}} F dx = \frac{A\left[\left(3.00^2\right) - (2.00)^2\right]}{2} - \frac{B\left[(3.00)^3 - (2.00)^3\right]}{3} = \boxed{\frac{5.00}{2}A - \frac{19.0}{3}B}$

$\Delta K = \boxed{\left(-\frac{5.00}{2}A + \frac{19.0}{3}B\right)}$

P8.41 (a) $W = \int F_x dx = \int_1^{5.00\text{ m}} (2x+4)dx = \left(\frac{2x^2}{2} + 4x\right)_1^{5.00\text{ m}} = 25.0 + 20.0 - 1.00 - 4.00 = \boxed{40.0\text{ J}}$

(b) $\Delta K + \Delta U = 0$ $\qquad \Delta U = -\Delta K = -W = \boxed{-40.0\text{ J}}$

(c) $\Delta K = K_f - \frac{mv_1^2}{2}$ $\qquad K_f = \Delta K + \frac{mv_1^2}{2} = \boxed{62.5\text{ J}}$

P8.42 $F_x = -\frac{\partial U}{\partial x} = -\frac{\partial\left(3x^3y - 7x\right)}{\partial x} = -\left(9x^2y - 7\right) = 7 - 9x^2y$

$F_y = -\frac{\partial U}{\partial y} = -\frac{\partial\left(3x^3y - 7x\right)}{\partial y} = -\left(3x^3 - 0\right) = -3x^3$

Thus, the force acting at the point $(x,\ y)$ is $\mathbf{F} = F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}} = \boxed{\left(7 - 9x^2y\right)\hat{\mathbf{i}} - 3x^3\hat{\mathbf{j}}}$.

P8.43 $U(r) = \frac{A}{r}$

$F_r = -\frac{\partial U}{\partial r} = -\frac{d}{dr}\left(\frac{A}{r}\right) = \boxed{\frac{A}{r^2}}$. The positive value indicates a force of repulsion.

Section 8.6 **Energy Diagrams and the Equilibrium of a System**

P8.44

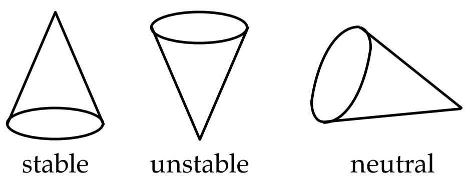

FIG. P8.44

P8.45 (a) F_x is zero at points A, C and E; F_x is positive at point B and negative at point D.

(b) A and E are unstable, and C is stable.

(c)

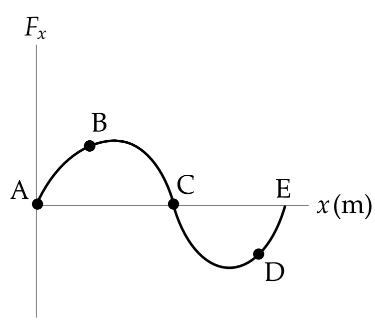

FIG. P8.45

P8.46 (a) There is an equilibrium point wherever the graph of potential energy is horizontal:

At $r = 1.5$ mm and 3.2 mm, the equilibrium is stable.
At $r = 2.3$ mm, the equilibrium is unstable.
A particle moving out toward $r \to \infty$ approaches neutral equilibrium.

(b) The system energy E cannot be less than –5.6 J. The particle is bound if $\boxed{-5.6\text{ J} \le E < 1\text{ J}}$.

(c) If the system energy is –3 J, its potential energy must be less than or equal to –3 J. Thus, the particle's position is limited to $\boxed{0.6\text{ mm} \le r \le 3.6\text{ mm}}$.

(d) $K + U = E$. Thus, $K_{max} = E - U_{min} = -3.0\text{ J} - (-5.6\text{ J}) = \boxed{2.6\text{ J}}$.

(e) Kinetic energy is a maximum when the potential energy is a minimum, at $\boxed{r = 1.5\text{ mm}}$.

(f) $-3\text{ J} + W = 1\text{ J}$. Hence, the binding energy is $W = \boxed{4\text{ J}}$.

P8.47 (a) When the mass moves distance x, the length of each spring changes from L to $\sqrt{x^2 + L^2}$, so each exerts force $k\left(\sqrt{x^2+L^2} - L\right)$ towards its fixed end. The y-components cancel out and the x components add to:

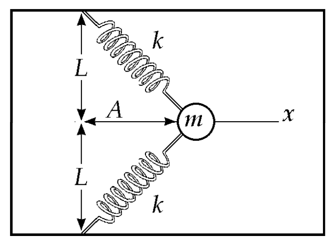

FIG. P8.47(a)

$$F_x = -2k\left(\sqrt{x^2+L^2} - L\right)\left(\frac{x}{\sqrt{x^2+L^2}}\right) = -2kx + \frac{2kLx}{\sqrt{x^2+L^2}}$$

Choose $U = 0$ at $x = 0$. Then at any point the potential energy of the system is

$$U(x) = -\int_0^x F_x dx = -\int_0^x \left(-2kx + \frac{2kLx}{\sqrt{x^2+L^2}}\right)dx = 2k\int_0^x x dx - 2kL\int_0^x \frac{x}{\sqrt{x^2+L^2}}dx$$

$$U(x) = \boxed{kx^2 + 2kL\left(L - \sqrt{x^2+L^2}\right)}$$

(b) $U(x) = 40.0x^2 + 96.0\left(1.20 - \sqrt{x^2 + 1.44}\right)$

For negative x, $U(x)$ has the same value as for positive x. The only equilibrium point (i.e., where $F_x = 0$) is $\boxed{x = 0}$.

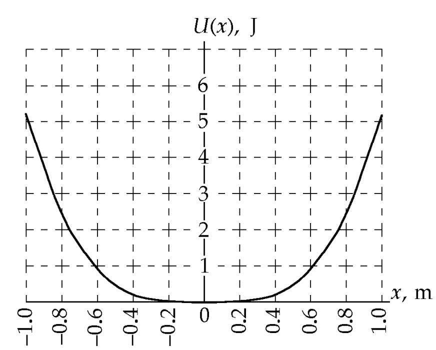

FIG. P8.47(b)

(c) $K_i + U_i + \Delta E_{mech} = K_f + U_f$

$0 + 0.400\text{ J} + 0 = \frac{1}{2}(1.18\text{ kg})v_f^2 + 0$

$v_f = \boxed{0.823\text{ m/s}}$

Additional Problems

P8.48 The potential energy of the block-Earth system is mgh.

An amount of energy $\mu_k mgd\cos\theta$ is converted into internal energy due to friction on the incline. Therefore the final height $y_{\max}$ is found from

$$mgy_{\max} = mgh - \mu_k mgd\cos\theta$$

where

$$d = \frac{y_{\max}}{\sin\theta}$$

$$\therefore mgy_{\max} = mgh - \mu_k mgy_{\max}\cot\theta$$

Solving,

$$y_{\max} = \frac{h}{1+\mu_k\cot\theta}.$$

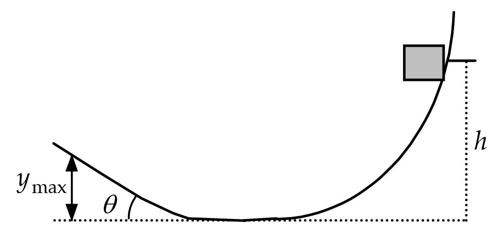

FIG. P8.48

P8.49 At a pace I could keep up for a half-hour exercise period, I climb two stories up, traversing forty steps each 18 cm high, in 20 s. My output work becomes the final gravitational energy of the system of the Earth and me,

$$mgy = (85\text{ kg})\left(9.80\text{ m/s}^2\right)(40\times 0.18\text{ m}) = 6\,000\text{ J}$$

making my sustainable power $\dfrac{6\,000\text{ J}}{20\text{ s}} = \boxed{\sim 10^2\text{ W}}$.

P8.50 $v = 100\text{ km/h} = 27.8\text{ m/s}$

The retarding force due to air resistance is

$$R = \frac{1}{2}D\rho Av^2 = \frac{1}{2}(0.330)\left(1.20\text{ kg/m}^3\right)\left(2.50\text{ m}^2\right)(27.8\text{ m/s})^2 = 382\text{ N}$$

Comparing the energy of the car at two points along the hill,

$$K_i + U_{gi} + \Delta E = K_f + U_{gf}$$

or $$K_i + U_{gi} + \Delta W_e - R(\Delta s) = K_f + U_{gf}$$

where ΔW_e is the work input from the engine. Thus,

$$\Delta W_e = R(\Delta s) + \left(K_f - K_i\right) + \left(U_{gf} - U_{gi}\right)$$

Recognizing that $K_f = K_i$ and dividing by the travel time Δt gives the required power input from the engine as

$$\mathcal{P} = \left(\frac{\Delta W_e}{\Delta t}\right) = R\left(\frac{\Delta s}{\Delta t}\right) + mg\left(\frac{\Delta y}{\Delta t}\right) = Rv + mgv\sin\theta$$

$$\mathcal{P} = (382\text{ N})(27.8\text{ m/s}) + (1\,500\text{ kg})\left(9.80\text{ m/s}^2\right)(27.8\text{ m/s})\sin 3.20°$$

$$\mathcal{P} = \boxed{33.4\text{ kW} = 44.8\text{ hp}}$$

P8.51 m = mass of pumpkin
R = radius of silo top

$$\sum F_r = ma_r \Rightarrow n - mg\cos\theta = -m\frac{v^2}{R}$$

When the pumpkin first loses contact with the surface, $n = 0$.
Thus, at the point where it leaves the surface: $v^2 = Rg\cos\theta$.

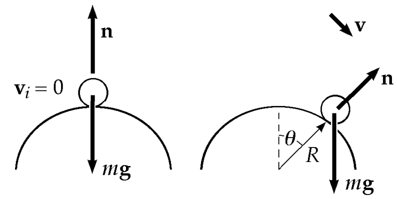

FIG. P8.51

Choose $U_g = 0$ in the $\theta = 90.0°$ plane. Then applying conservation of energy for the pumpkin-Earth system between the starting point and the point where the pumpkin leaves the surface gives

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}mv^2 + mgR\cos\theta = 0 + mgR$$

Using the result from the force analysis, this becomes

$\frac{1}{2}mRg\cos\theta + mgR\cos\theta = mgR$, which reduces to

$\cos\theta = \frac{2}{3}$, and gives $\theta = \cos^{-1}(2/3) = \boxed{48.2°}$

as the angle at which the pumpkin will lose contact with the surface.

P8.52 (a) $U_A = mgR = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.300\text{ m}) = \boxed{0.588\text{ J}}$

(b) $K_A + U_A = K_B + U_B$
$K_B = K_A + U_A - U_B = mgR = \boxed{0.588\text{ J}}$

(c) $v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{2(0.588\text{ J})}{0.200\text{ kg}}} = \boxed{2.42\text{ m/s}}$

(d) $U_C = mgh_C = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m}) = \boxed{0.392\text{ J}}$
$K_C = K_A + U_A - U_C = mg(h_A - h_C)$
$K_C = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.300 - 0.200)\text{ m} = \boxed{0.196\text{ J}}$

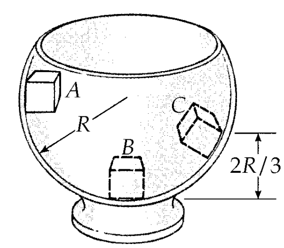

FIG. P8.52

P8.53 (a) $K_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.200\text{ kg})(1.50\text{ m/s})^2 = \boxed{0.225\text{ J}}$

(b)
$$\begin{aligned}\Delta E_{\text{mech}} &= \Delta K + \Delta U = K_B - K_A + U_B - U_A \\ &= K_B + mg(h_B - h_A) \\ &= 0.225\text{ J} + (0.200\text{ kg})(9.80\text{ m/s}^2)(0 - 0.300\text{ m}) \\ &= 0.225\text{ J} - 0.588\text{ J} = \boxed{-0.363\text{ J}}\end{aligned}$$

(c) It's possible to find an effective coefficient of friction, but not the actual value of μ since n and f vary with position.

P8.54 The gain in internal energy due to friction represents a loss in mechanical energy that must be equal to the change in the kinetic energy plus the change in the potential energy.

Therefore,

$$-\mu_k mgx\cos\theta = \Delta K + \frac{1}{2}kx^2 - mgx\sin\theta$$

and since $v_i = v_f = 0$, $\Delta K = 0$.

Thus,

$$-\mu_k(2.00)(9.80)(\cos 37.0°)(0.200) = \frac{(100)(0.200)^2}{2} - (2.00)(9.80)(\sin 37.0°)(0.200)$$

and we find $\mu_k = \boxed{0.115}$. Note that in the above we had a *gain* in elastic potential energy for the spring and a *loss* in gravitational potential energy.

P8.55 (a) Since no nonconservative work is done, $\Delta E = 0$

Also $\Delta K = 0$

therefore, $U_i = U_f$

where $U_i = (mg\sin\theta)x$

and $U_f = \frac{1}{2}kx^2$

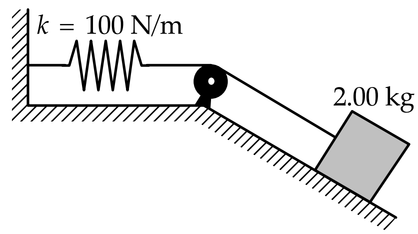

FIG. P8.55

Substituting values yields $(2.00)(9.80)\sin 37.0° = (100)\frac{x}{2}$ and solving we find

$$x = \boxed{0.236\text{ m}}$$

(b) $\sum F = ma$. Only gravity and the spring force act on the block, so

$$-kx + mg\sin\theta = ma$$

For $x = 0.236$ m,

$a = \boxed{-5.90\text{ m/s}^2}$. The negative sign indicates a is up the incline.

The $\boxed{\text{acceleration depends on position}}$.

(c) U(gravity) decreases monotonically as the height decreases.

U(spring) increases monotonically as the spring is stretched.

K initially increases, but then goes back to zero.

P8.56 $k = 2.50 \times 10^4 \text{ N/m}$, $m = 25.0 \text{ kg}$

$x_A = -0.100 \text{ m}$, $U_g\big|_{x=0} = U_s\big|_{x=0} = 0$

(a) $E_{\text{mech}} = K_A + U_{gA} + U_{sA}$

$$E_{\text{mech}} = 0 + mgx_A + \frac{1}{2}kx_A^2$$
$$E_{\text{mech}} = (25.0 \text{ kg})(9.80 \text{ m/s}^2)(-0.100 \text{ m}) + \frac{1}{2}(2.50 \times 10^4 \text{ N/m})(-0.100 \text{ m})^2$$
$$E_{\text{mech}} = -24.5 \text{ J} + 125 \text{ J} = \boxed{100 \text{ J}}$$

(b) Since only conservative forces are involved, the total energy of the child-pogo-stick-Earth system at point *C* is the same as that at point *A*.

$K_C + U_{gC} + U_{sC} = K_A + U_{gA} + U_{sA}$: $\quad 0 + (25.0 \text{ kg})(9.80 \text{ m/s}^2)x_C + 0 = 0 - 24.5 \text{ J} + 125 \text{ J}$

$$x_C = \boxed{0.410 \text{ m}}$$

(c) $K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$: $\quad \frac{1}{2}(25.0 \text{ kg})v_B^2 + 0 + 0 = 0 + (-24.5 \text{ J}) + 125 \text{ J}$

$$v_B = \boxed{2.84 \text{ m/s}}$$

(d) *K* and *v* are at a maximum when $a = \sum F/m = 0$ (i.e., when the magnitude of the upward spring force equals the magnitude of the downward gravitational force).

This occurs at $x < 0$ where $k|x| = mg$

or $$|x| = \frac{(25.0 \text{ kg})(9.8 \text{ m/s}^2)}{2.50 \times 10^4 \text{ N/m}} = 9.80 \times 10^{-3} \text{ m}$$

Thus, $K = K_{\max}$ at $x = \boxed{-9.80 \text{ mm}}$

(e) $$K_{\max} = K_A + \left(U_{gA} - U_g\big|_{x=-9.80 \text{ mm}}\right) + \left(U_{sA} - U_s\big|_{x=-9.80 \text{ mm}}\right)$$

or $$\frac{1}{2}(25.0 \text{ kg})v_{\max}^2 = (25.0 \text{ kg})(9.80 \text{ m/s}^2)\left[(-0.100 \text{ m}) - (-0.009\,8 \text{ m})\right] + \frac{1}{2}(2.50 \times 10^4 \text{ N/m})\left[(-0.100 \text{ m})^2 - (-0.009\,8 \text{ m})^2\right]$$

yielding $v_{\max} = \boxed{2.85 \text{ m/s}}$

P8.57 $\Delta E_{\text{mech}} = -f\Delta x$

$$E_f - E_i = -f \cdot d_{BC}$$
$$\frac{1}{2}kx^2 - mgh = -\mu mgd_{BC}$$
$$\mu = \frac{mgh - \frac{1}{2}kx^2}{mgd_{BC}} = \boxed{0.328}$$

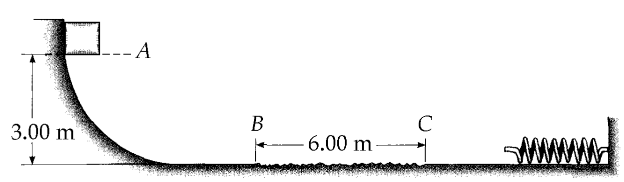

FIG. P8.57

P8.58 (a) $\mathbf{F} = -\frac{d}{dx}\left(-x^3 + 2x^2 + 3x\right)\hat{\mathbf{i}} = \boxed{\left(3x^2 - 4x - 3\right)\hat{\mathbf{i}}}$

(b) $F = 0$

when $x = \boxed{1.87 \text{ and } -0.535}$

(c) The stable point is at

$$x = -0.535 \text{ point of minimum } U(x).$$

The unstable point is at

$$x = 1.87 \text{ maximum in } U(x).$$

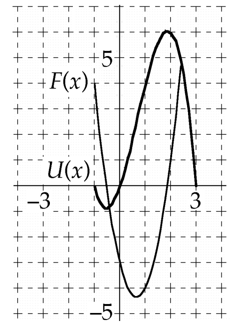

FIG. P8.58

P8.59 $(K+U)_i = (K+U)_f$

$0 + (30.0 \text{ kg})\left(9.80 \text{ m/s}^2\right)(0.200 \text{ m}) + \frac{1}{2}(250 \text{ N/m})(0.200 \text{ m})^2$

$= \frac{1}{2}(50.0 \text{ kg})v^2 + (20.0 \text{ kg})\left(9.80 \text{ m/s}^2\right)(0.200 \text{ m})\sin 40.0°$

$58.8 \text{ J} + 5.00 \text{ J} = (25.0 \text{ kg})v^2 + 25.2 \text{ J}$

$\boxed{v = 1.24 \text{ m/s}}$

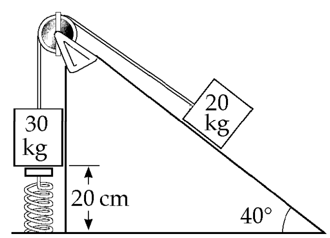

FIG. P8.59

P8.60 (a) Between the second and the third picture, $\Delta E_{\text{mech}} = \Delta K + \Delta U$

$$-\mu mgd = -\frac{1}{2}mv_i^2 + \frac{1}{2}kd^2$$

$$\frac{1}{2}(50.0 \text{ N/m})d^2 + 0.250(1.00 \text{ kg})\left(9.80 \text{ m/s}^2\right)d - \frac{1}{2}(1.00 \text{ kg})\left(3.00 \text{ m/s}^2\right) = 0$$

$$d = \frac{[-2.45 \pm 21.25] \text{ N}}{50.0 \text{ N/m}} = \boxed{0.378 \text{ m}}$$

(b) Between picture two and picture four, $\Delta E_{\text{mech}} = \Delta K + \Delta U$

$$-f(2d) = \frac{1}{2}mv^2 - \frac{1}{2}mv_i^2$$

$$v = \sqrt{(3.00 \text{ m/s})^2 - \frac{2}{(1.00 \text{ kg})}(2.45 \text{ N})(2)(0.378 \text{ m})}$$

$$= \boxed{2.30 \text{ m/s}}$$

(c) For the motion from picture two to picture five, $\Delta E_{\text{mech}} = \Delta K + \Delta U$

$$-f(D + 2d) = -\frac{1}{2}(1.00 \text{ kg})(3.00 \text{ m/s})^2$$

$$D = \frac{9.00 \text{ J}}{2(0.250)(1.00 \text{ kg})\left(9.80 \text{ m/s}^2\right)} - 2(0.378 \text{ m}) = \boxed{1.08 \text{ m}}$$

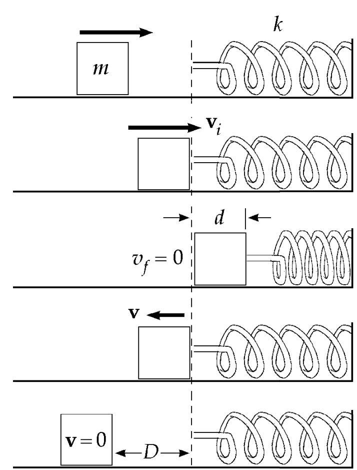

FIG. P8.60

P8.61 (a) Initial compression of spring: $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

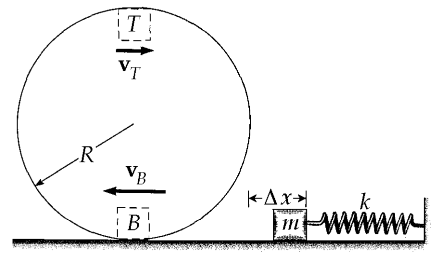

FIG. P8.61

$$\frac{1}{2}(450\ \text{N/m})(\Delta x)^2 = \frac{1}{2}(0.500\ \text{kg})(12.0\ \text{m/s})^2$$

$$\therefore \Delta x = \boxed{0.400\ \text{m}}$$

(b) Speed of block at top of track: $\Delta E_{\text{mech}} = -f\Delta x$

$$\left(mgh_T + \frac{1}{2}mv_T^2\right) - \left(mgh_B + \frac{1}{2}mv_B^2\right) = -f(\pi R)$$

$$(0.500\ \text{kg})\left(9.80\ \text{m/s}^2\right)(2.00\ \text{m}) + \frac{1}{2}(0.500\ \text{kg})v_T^2 - \frac{1}{2}(0.500\ \text{kg})(12.0\ \text{m/s})^2$$
$$= -(7.00\ \text{N})(\pi)(1.00\ \text{m})$$

$$0.250v_T^2 = 4.21$$

$$\therefore v_T = \boxed{4.10\ \text{m/s}}$$

(c) Does block fall off at or before top of track? Block falls if $a_c < g$

$$a_c = \frac{v_T^2}{R} = \frac{(4.10)^2}{1.00} = 16.8\ \text{m/s}^2$$

Therefore $a_c > g$ and the $\boxed{\text{block stays on the track}}$.

P8.62 Let λ represent the mass of each one meter of the chain and T represent the tension in the chain at the table edge. We imagine the edge to act like a frictionless and massless pulley.

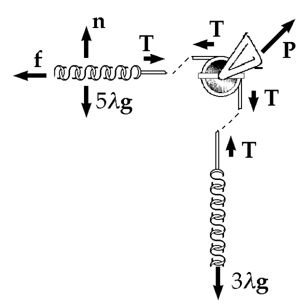

FIG. P8.62

(a) For the five meters on the table with motion impending,

$\sum F_y = 0: \quad +n - 5\lambda g = 0 \qquad n = 5\lambda g$

$f_s \le \mu_s n = 0.6(5\lambda g) = 3\lambda g$

$\sum F_x = 0: \quad +T - f_s = 0 \qquad T = f_s \qquad T \le 3\lambda g$

The maximum value is barely enough to support the hanging segment according to

$\sum F_y = 0: \quad +T - 3\lambda g = 0 \qquad T = 3\lambda g$

so it is at this point that the chain starts to slide.

continued on next page

(b) Let x represent the variable distance the chain has slipped since the start.

Then length $(5-x)$ remains on the table, with now

$$\sum F_y = 0: \quad +n-(5-x)\lambda g = 0 \qquad n = (5-x)\lambda g$$
$$f_k = \mu_k n = 0.4(5-x)\lambda g = 2\lambda g - 0.4x\lambda g$$

Consider energies of the chain-Earth system at the initial moment when the chain starts to slip, and a final moment when $x = 5$, when the last link goes over the brink. Measure heights above the final position of the leading end of the chain. At the moment the final link slips off, the center of the chain is at $y_f = 4$ meters.

Originally, 5 meters of chain is at height 8 m and the middle of the dangling segment is at height $8 - \frac{3}{2} = 6.5$ m.

$$K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f: \qquad 0 + (m_1 g y_1 + m_2 g y_2)_i - \int_i^f f_k dx = \left(\frac{1}{2}mv^2 + mgy\right)_f$$

$$(5\lambda g)8 + (3\lambda g)6.5 - \int_0^5 (2\lambda g - 0.4x\lambda g)dx = \frac{1}{2}(8\lambda)v^2 + (8\lambda g)4$$

$$40.0g + 19.5g - 2.00g\int_0^5 dx + 0.400g\int_0^5 x\,dx = 4.00v^2 + 32.0g$$

$$27.5g - 2.00gx\Big|_0^5 + 0.400g\frac{x^2}{2}\Big|_0^5 = 4.00v^2$$

$$27.5g - 2.00g(5.00) + 0.400g(12.5) = 4.00v^2$$

$$22.5g = 4.00v^2$$

$$v = \sqrt{\frac{(22.5\text{ m})(9.80\text{ m/s}^2)}{4.00}} = \boxed{7.42\text{ m/s}}$$

P8.63 Launch speed is found from

$$mg\left(\frac{4}{5}h\right) = \frac{1}{2}mv^2: \qquad v = \sqrt{2g\left(\frac{4}{5}\right)h}$$
$$v_y = v\sin\theta$$

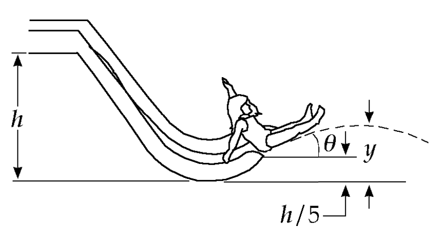

FIG. P8.63

The height y above the water (by conservation of energy for the child-Earth system) is found from

$$mgy = \frac{1}{2}mv_y^2 + mg\frac{h}{5} \qquad \left(\text{since } \frac{1}{2}mv_x^2 \text{ is constant in projectile motion}\right)$$

$$y = \frac{1}{2g}v_y^2 + \frac{h}{5} = \frac{1}{2g}v^2\sin^2\theta + \frac{h}{5}$$

$$y = \frac{1}{2g}\left[2g\left(\frac{4}{5}h\right)\right]\sin^2\theta + \frac{h}{5} = \boxed{\frac{4}{5}h\sin^2\theta + \frac{h}{5}}$$

***P8.64** (a) The length of string between glider and pulley is given by $\ell^2 = x^2 + h_0^2$. Then $2\ell\frac{d\ell}{dt} = 2x\frac{dx}{dt} + 0$.

Now $\frac{d\ell}{dt}$ is the rate at which string goes over the pulley: $\frac{d\ell}{dt} = v_y = \frac{x}{\ell}v_x = (\cos\theta)v_x$.

(b) $(K_A + K_B + U_g)_i = (K_A + K_B + U_g)_f$

$0 + 0 + m_B g(y_{30} - y_{45}) = \frac{1}{2}m_A v_x^2 + \frac{1}{2}m_B v_y^2$

Now $y_{30} - y_{45}$ is the amount of string that has gone over the pulley, $\ell_{30} - \ell_{45}$. We have $\sin 30° = \frac{h_0}{\ell_{30}}$ and $\sin 45° = \frac{h_0}{\ell_{45}}$, so $\ell_{30} - \ell_{45} = \frac{h_0}{\sin 30°} - \frac{h_0}{\sin 45°} = 0.40\text{ m}\left(2 - \sqrt{2}\right) = 0.234\text{ m}$.

From the energy equation

$$0.5\text{ kg } 9.8\text{ m/s}^2\ 0.234\text{ m} = \frac{1}{2}1.00\text{ kg } v_x^2 + \frac{1}{2}0.500\text{ kg } v_x^2\cos^2 45°$$

$$v_x = \sqrt{\frac{1.15\text{ J}}{0.625\text{ kg}}} = \boxed{1.35\text{ m/s}}$$

(c) $v_y = v_x\cos\theta = (1.35\text{ m/s})\cos 45° = \boxed{0.958\text{ m/s}}$

(d) The acceleration of neither glider is constant, so knowing distance and acceleration at one point is not sufficient to find speed at another point.

P8.65 The geometry reveals $D = L\sin\theta + L\sin\phi$, $50.0\text{ m} = 40.0\text{ m}(\sin 50° + \sin\phi)$, $\phi = 28.9°$

(a) From takeoff to alighting for the Jane-Earth system

$$(K + U_g)_i + W_{\text{wind}} = (K + U_g)_f$$

$$\frac{1}{2}mv_i^2 + mg(-L\cos\theta) + FD(-1) = 0 + mg(-L\cos\phi)$$

$$\frac{1}{2}50\text{ kg } v_i^2 + 50\text{ kg}\left(9.8\text{ m/s}^2\right)(-40\text{ m}\cos 50°) - 110\text{ N}(50\text{ m}) = 50\text{ kg}\left(9.8\text{ m/s}^2\right)(-40\text{ m}\cos 28.9°)$$

$$\frac{1}{2}50\text{ kg } v_i^2 - 1.26\times 10^4\text{ J} - 5.5\times 10^3\text{ J} = -1.72\times 10^4\text{ J}$$

$$v_i = \sqrt{\frac{2(947\text{ J})}{50\text{ kg}}} = \boxed{6.15\text{ m/s}}$$

(b) For the swing back

$$\frac{1}{2}mv_i^2 + mg(-L\cos\phi) + FD(+1) = 0 + mg(-L\cos\theta)$$

$$\frac{1}{2}130\text{ kg } v_i^2 + 130\text{ kg}\left(9.8\text{ m/s}^2\right)(-40\text{ m}\cos 28.9°) + 110\text{ N}(50\text{ m})$$

$$= 130\text{ kg}\left(9.8\text{ m/s}^2\right)(-40\text{ m}\cos 50°)$$

$$\frac{1}{2}130\text{ kg } v_i^2 - 4.46\times 10^4\text{ J} + 5\,500\text{ J} = -3.28\times 10^4\text{ J}$$

$$v_i = \sqrt{\frac{2(6\,340\text{ J})}{130\text{ kg}}} = \boxed{9.87\text{ m/s}}$$

P8.66 Case I: Surface is frictionless

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$k = \frac{mv^2}{x^2} = \frac{(5.00\ \text{kg})(1.20\ \text{m/s})^2}{10^{-2}\ \text{m}^2} = 7.20 \times 10^2\ \text{N/m}$$

Case II: Surface is rough, $\mu_k = 0.300$

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 - \mu_k mgx$$

$$\frac{5.00\ \text{kg}}{2}v^2 = \frac{1}{2}\left(7.20 \times 10^2\ \text{N/m}\right)\left(10^{-1}\ \text{m}\right)^2 - (0.300)(5.00\ \text{kg})\left(9.80\ \text{m/s}^2\right)\left(10^{-1}\ \text{m}\right)$$

$$\boxed{v = 0.923\ \text{m/s}}$$

***P8.67** (a) $\left(K + U_g\right)_A = \left(K + U_g\right)_B$

$$0 + mgy_A = \frac{1}{2}mv_B^2 + 0 \qquad v_B = \sqrt{2gy_A} = \sqrt{2\left(9.8\ \text{m/s}^2\right)6.3\ \text{m}} = \boxed{11.1\ \text{m/s}}$$

(b) $$a_c = \frac{v^2}{r} = \frac{(11.1\ \text{m/s})^2}{6.3\ \text{m}} = \boxed{19.6\ \text{m/s}^2\ \text{up}}$$

(c) $\sum F_y = ma_y \qquad +n_B - mg = ma_c$

$$n_B = 76\ \text{kg}\left(9.8\ \text{m/s}^2 + 19.6\ \text{m/s}^2\right) = \boxed{2.23 \times 10^3\ \text{N up}}$$

(d) $$W = F\Delta r\cos\theta = 2.23 \times 10^3\ \text{N}(0.450\ \text{m})\cos 0° = \boxed{1.01 \times 10^3\ \text{J}}$$

(e) $\left(K + U_g\right)_B + W = \left(K + U_g\right)_D$

$$\frac{1}{2}mv_B^2 + 0 + 1.01 \times 10^3\ \text{J} = \frac{1}{2}mv_D^2 + mg(y_D - y_B)$$

$$\frac{1}{2}76\ \text{kg}(11.1\ \text{m/s})^2 + 1.01 \times 10^3\ \text{J} = \frac{1}{2}76\ \text{kg}\ v_D^2 + 76\ \text{kg}\left(9.8\ \text{m/s}^2\right)6.3\ \text{m}$$

$$\sqrt{\frac{\left(5.70 \times 10^3\ \text{J} - 4.69 \times 10^3\ \text{J}\right)2}{76\ \text{kg}}} = v_D = \boxed{5.14\ \text{m/s}}$$

(f) $\left(K + U_g\right)_D = \left(K + U_g\right)_E$ where E is the apex of his motion

$$\frac{1}{2}mv_D^2 + 0 = 0 + mg(y_E - y_D) \qquad y_E - y_D = \frac{v_D^2}{2g} = \frac{(5.14\ \text{m/s})^2}{2\left(9.8\ \text{m/s}^2\right)} = \boxed{1.35\ \text{m}}$$

(g) Consider the motion with constant acceleration between takeoff and touchdown. The time is the positive root of

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$-2.34\ \text{m} = 0 + 5.14\ \text{m/s}\,t + \frac{1}{2}\left(-9.8\ \text{m/s}^2\right)t^2$$

$$4.9t^2 - 5.14t - 2.34 = 0$$

$$t = \frac{5.14 \pm \sqrt{5.14^2 - 4(4.9)(-2.34)}}{9.8} = \boxed{1.39\ \text{s}}$$

***P8.68** If the spring is just barely able to lift the lower block from the table, the spring lifts it through no noticeable distance, but exerts on the block a force equal to its weight Mg. The extension of the spring, from $|\mathbf{F}_s| = kx$, must be Mg/k. Between an initial point at release and a final point when the moving block first comes to rest, we have

$$K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}: \qquad 0 + mg\left(-\frac{4mg}{k}\right) + \frac{1}{2}k\left(\frac{4mg}{k}\right)^2 = 0 + mg\left(\frac{Mg}{k}\right) + \frac{1}{2}k\left(\frac{Mg}{k}\right)^2$$

$$-\frac{4m^2g^2}{k} + \frac{8m^2g^2}{k} = \frac{mMg^2}{k} + \frac{M^2g^2}{2k}$$

$$4m^2 = mM + \frac{M^2}{2}$$

$$\frac{M^2}{2} + mM - 4m^2 = 0$$

$$M = \frac{-m \pm \sqrt{m^2 - 4\left(\frac{1}{2}\right)\left(-4m^2\right)}}{2\left(\frac{1}{2}\right)} = -m \pm \sqrt{9m^2}$$

Only a positive mass is physical, so we take $M = m(3-1) = \boxed{2m}$.

P8.69 (a) Take the original point where the ball is released and the final point where its upward swing stops at height H and horizontal displacement

$$x = \sqrt{L^2 - (L-H)^2} = \sqrt{2LH - H^2}$$

Since the wind force is purely horizontal, it does work

$$W_{\text{wind}} = \int \mathbf{F} \cdot d\mathbf{s} = F\int dx = F\sqrt{2LH - H^2}$$

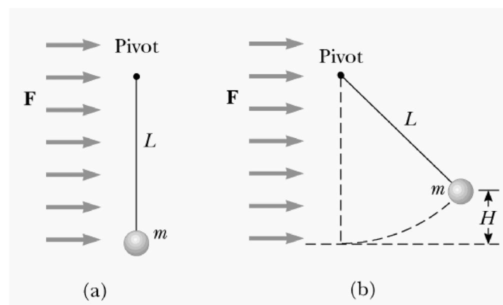

FIG. P8.69

The work-energy theorem can be written:

$K_i + U_{gi} + W_{\text{wind}} = K_f + U_{gf}$, or

$0 + 0 + F\sqrt{2LH - H^2} = 0 + mgH$ giving $F^2 2LH - F^2H^2 = m^2g^2H^2$

Here $H = 0$ represents the lower turning point of the ball's oscillation, and the upper limit is at $F^2(2L) = \left(F^2 + m^2g^2\right)H$. Solving for H yields

$$H = \frac{2LF^2}{F^2 + m^2g^2} = \boxed{\frac{2L}{1 + (mg/F)^2}}$$

As $F \to 0$, $H \to 0$ as is reasonable.
As $F \to \infty$, $H \to 2L$, which would be hard to approach experimentally.

(b) $$H = \frac{2(2.00\text{ m})}{1 + \left[(2.00\text{ kg})\left(9.80\text{ m/s}^2\right)/14.7\text{ N}\right]^2} = \boxed{1.44\text{ m}}$$

continued on next page

(c) Call θ the equilibrium angle with the vertical.

$$\sum F_x = 0 \Rightarrow T\sin\theta = F, \text{ and}$$
$$\sum F_y = 0 \Rightarrow T\cos\theta = mg$$

Dividing: $\tan\theta = \frac{F}{mg} = \frac{14.7 \text{ N}}{19.6 \text{ N}} = 0.750$, or $\theta = 36.9°$

Therefore, $H_{eq} = L(1-\cos\theta) = (2.00 \text{ m})(1-\cos 36.9°) = \boxed{0.400 \text{ m}}$

(d) As $F \to \infty$, $\tan\theta \to \infty$, $\theta \to 90.0°$ and $H_{eq} \to L$

A very strong wind pulls the string out horizontal, parallel to the ground. Thus, $\boxed{\left(H_{eq}\right)_{max} = L}$.

P8.70 Call $\phi = 180° - \theta$ the angle between the upward vertical and the radius to the release point. Call v_r the speed here. By conservation of energy

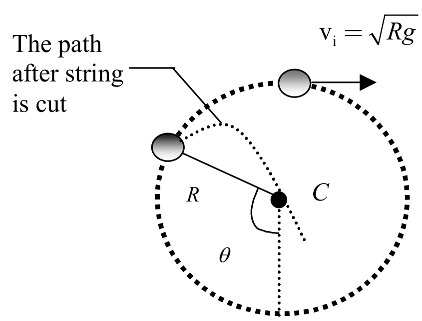

FIG. P8.70

$$K_i + U_i + \Delta E = K_r + U_r$$
$$\frac{1}{2}mv_i^2 + mgR + 0 = \frac{1}{2}mv_r^2 + mgR\cos\phi$$
$$gR + 2gR = v_r^2 + 2gR\cos\phi$$
$$v_r = \sqrt{3gR - 2gR\cos\phi}$$

The components of velocity at release are $v_x = v_r\cos\phi$ and $v_y = v_r\sin\phi$ so for the projectile motion we have

$$x = v_x t \qquad R\sin\phi = v_r\cos\phi t$$
$$y = v_y t - \frac{1}{2}gt^2 \qquad -R\cos\phi = v_r\sin\phi t - \frac{1}{2}gt^2$$

By substitution

$$-R\cos\phi = v_r\sin\phi\frac{R\sin\phi}{v_r\cos\phi} - \frac{g}{2}\frac{R^2\sin^2\phi}{v_r^2\cos^2\phi}$$

with $\sin^2\phi + \cos^2\phi = 1$,

$$gR\sin^2\phi = 2v_r^2\cos\phi = 2\cos\phi(3gR - 2gR\cos\phi)$$
$$\sin^2\phi = 6\cos\phi - 4\cos^2\phi = 1 - \cos^2\phi$$
$$3\cos^2\phi - 6\cos\phi + 1 = 0$$
$$\cos\phi = \frac{6 \pm \sqrt{36-12}}{6}$$

Only the – sign gives a value for $\cos\phi$ that is less than one:

$$\cos\phi = 0.183\,5 \qquad \phi = 79.43° \qquad \text{so } \theta = \boxed{100.6°}$$

P8.71 Applying Newton's second law at the bottom (b) and top (t) of the circle gives

$$T_b - mg = \frac{mv_b^2}{R} \text{ and } -T_t - mg = -\frac{mv_t^2}{R}$$

Adding these gives $T_b = T_t + 2mg + \frac{m\left(v_b^2 - v_t^2\right)}{R}$

Also, energy must be conserved and $\Delta U + \Delta K = 0$

So, $\frac{m\left(v_b^2 - v_t^2\right)}{2} + (0 - 2mgR) = 0$ and $\frac{m\left(v_b^2 - v_t^2\right)}{R} = 4mg$

Substituting into the above equation gives $\boxed{T_b = T_t + 6mg}$.

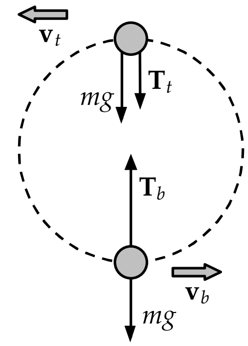

FIG. P8.71

P8.72 (a) Energy is conserved in the swing of the pendulum, and the stationary peg does no work. So the ball's speed does not change when the string hits or leaves the peg, and the ball swings equally high on both sides.

(b) Relative to the point of suspension,

$$U_i = 0,\, U_f = -mg\left[d - (L - d)\right]$$

From this we find that

$$-mg(2d - L) + \frac{1}{2}mv^2 = 0$$

Also for centripetal motion,

$$mg = \frac{mv^2}{R} \text{ where } R = L - d.$$

Upon solving, we get $\boxed{d = \frac{3L}{5}}$.

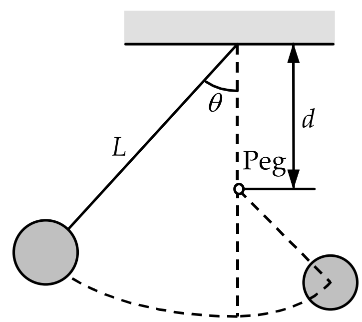

FIG. P8.72

***P8.73** (a) At the top of the loop the car and riders are in free fall:

$$\sum F_y = ma_y: \qquad mg \text{ down} = \frac{mv^2}{R} \text{ down}$$

$$v = \sqrt{Rg}$$

Energy of the car-riders-Earth system is conserved between release and top of loop:

$$K_i + U_{gi} = K_f + U_{gf}: \qquad 0 + mgh = \frac{1}{2}mv^2 + mg(2R)$$

$$gh = \frac{1}{2}Rg + g(2R)$$

$$\boxed{h = 2.50R}$$

(b) Let h now represent the height $\geq 2.5R$ of the release point. At the bottom of the loop we have

$$mgh = \frac{1}{2}mv_b^2 \qquad \text{or} \qquad v_b^2 = 2gh$$

$$\sum F_y = ma_y: \qquad n_b - mg = \frac{mv_b^2}{R}(\text{up})$$

$$n_b = mg + \frac{m(2gh)}{R}$$

At the top of the loop, $mgh = \frac{1}{2}mv_t^2 + mg(2R)$

$$v_t^2 = 2gh - 4gR$$

$$\sum F_y = ma_y: \qquad -n_t - mg = -\frac{mv_t^2}{R}$$

$$n_t = -mg + \frac{m}{R}(2gh - 4gR)$$

$$n_t = \frac{m(2gh)}{R} - 5mg$$

Then the normal force at the bottom is larger by

$$n_b - n_t = mg + \frac{m(2gh)}{R} - \frac{m(2gh)}{R} + 5mg = \boxed{6mg}.$$

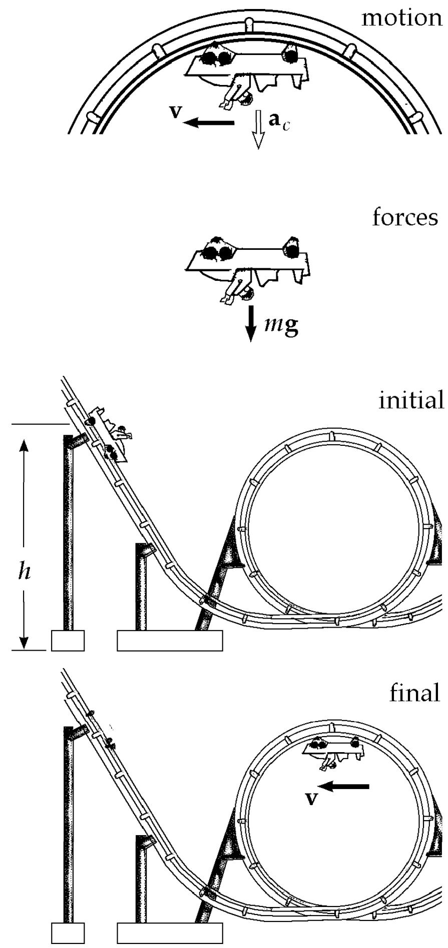

FIG. P8.73

***P8.74** (a) Conservation of energy for the sled-rider-Earth system, between A and C:

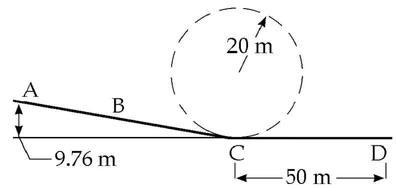

FIG. P8.74(a)

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}m(2.5\ \text{m/s})^2 + m(9.80\ \text{m/s}^2)(9.76\ \text{m}) = \frac{1}{2}mv_C^2 + 0$$

$$v_C = \sqrt{(2.5\ \text{m/s})^2 + 2(9.80\ \text{m/s}^2)(9.76\ \text{m})} = \boxed{14.1\ \text{m/s}}$$

(b) Incorporating the loss of mechanical energy during the portion of the motion in the water, we have, for the entire motion between A and D (the rider's stopping point),

$$K_i + U_{gi} - f_k\Delta x = K_f + U_{gf}: \quad \frac{1}{2}(80\ \text{kg})(2.5\ \text{m/s})^2 + (80\ \text{kg})(9.80\ \text{m/s}^2)(9.76\ \text{m}) - f_k\Delta x = 0 + 0$$

$$-f_k\Delta x = \boxed{-7.90\times 10^3\ \text{J}}$$

(c) The water exerts a frictional force $f_k = \frac{7.90\times 10^3\ \text{J}}{\Delta x} = \frac{7.90\times 10^3\ \text{N}\cdot\text{m}}{50\ \text{m}} = 158\ \text{N}$

and also a normal force of $n = mg = (80\ \text{kg})(9.80\ \text{m/s}^2) = 784\ \text{N}$

The magnitude of the water force is $\sqrt{(158\ \text{N})^2 + (784\ \text{N})^2} = \boxed{800\ \text{N}}$

(d) The angle of the slide is

$$\theta = \sin^{-1}\frac{9.76\ \text{m}}{54.3\ \text{m}} = 10.4°$$

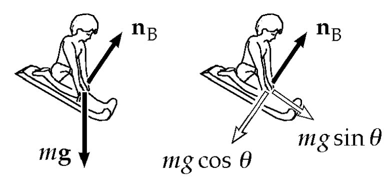

FIG. P8.74(d)

For forces perpendicular to the track at B,

$$\sum F_y = ma_y: \quad n_B - mg\cos\theta = 0$$

$$n_B = (80.0\ \text{kg})(9.80\ \text{m/s}^2)\cos 10.4° = \boxed{771\ \text{N}}$$

(e)

$$\sum F_y = ma_y: \quad +n_C - mg = \frac{mv_C^2}{r}$$

$$n_C = (80.0\ \text{kg})(9.80\ \text{m/s}^2) + \frac{(80.0\ \text{kg})(14.1\ \text{m/s})^2}{20\ \text{m}}$$

$$n_C = \boxed{1.57\times 10^3\ \text{N up}}$$

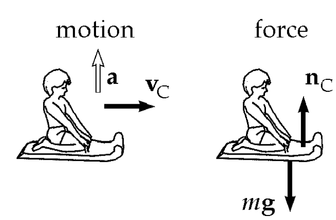

FIG. P8.74(e)

The rider pays for the thrills of a giddy height at A, and a high speed and tremendous splash at C. As a bonus, he gets the quick change in direction and magnitude among the forces we found in parts (d), (e), and (c).

ANSWERS TO EVEN PROBLEMS

P8.2 (a) 800 J; (b) 107 J; (c) 0

P8.4 (a) 1.11×10^9 J; (b) 0.2

P8.6 1.84 m

P8.8 (a) 10.2 kW; (b) 10.6 kW; (c) 5.82×10^6 J

P8.10 $d = \frac{kx^2}{2mg\sin\theta} - x$

P8.12 (a) see the solution; (b) 60.0°

P8.14 (a) $\sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)}}$; (b) $\frac{2m_1 h}{m_1 + m_2}$

P8.16 160 L/min

P8.18 40.8°

P8.20 $\left(\frac{8gh}{15}\right)^{1/2}$

P8.22 (a) see the solution; (b) 35.0 J

P8.24 (a) $v_B = 5.94$ m/s; $v_C = 7.67$ m/s; (b) 147 J

P8.26 (a) $U_f = 22.0$ J; $E = 40.0$ J; (b) Yes. The total mechanical energy changes.

P8.28 194 m

P8.30 2.06 kN up

P8.32 168 J

P8.34 (a) 24.5 m/s; (b) yes; (c) 206 m; (d) Air drag depends strongly on speed.

P8.36 3.92 kJ

P8.38 44.1 kW

P8.40 (a) $\frac{Ax^2}{2} - \frac{Bx^3}{3}$;
(b) $\Delta U = \frac{5A}{2} - \frac{19B}{3}$; $\Delta K = \frac{19B}{3} - \frac{5A}{2}$

P8.42 $(7 - 9x^2 y)\hat{\mathbf{i}} - 3x^3\hat{\mathbf{j}}$

P8.44 see the solution

P8.46 (a) $r = 1.5$ mm and 3.2 mm, stable; 2.3 mm and unstable; $r \to \infty$ neutral;
(b) $-5.6 \text{ J} \le E < 1 \text{ J}$; (c) $0.6 \text{ mm} \le r \le 3.6 \text{ mm}$;
(d) 2.6 J; (e) 1.5 mm; (f) 4 J

P8.48 see the solution

P8.50 33.4 kW

P8.52 (a) 0.588 J; (b) 0.588 J; (c) 2.42 m/s;
(d) 0.196 J; 0.392 J

P8.54 0.115

P8.56 (a) 100 J; (b) 0.410 m; (c) 2.84 m/s;
(d) −9.80 mm; (e) 2.85 m/s

P8.58 (a) $(3x^2 - 4x - 3)\hat{\mathbf{i}}$; (b) 1.87; −0.535;
(c) see the solution

P8.60 (a) 0.378 m; (b) 2.30 m/s; (c) 1.08 m

P8.62 (a) see the solution; (b) 7.42 m/s

P8.64 (a) see the solution; (b) 1.35 m/s;
(c) 0.958 m/s; (d) see the solution

P8.66 0.923 m/s

P8.68 $2m$

P8.70 100.6°

P8.72 see the solution

P8.74 (a) 14.1 m/s; (b) −7.90 J; (c) 800 N;
(d) 771 N; (e) 1.57 kN up